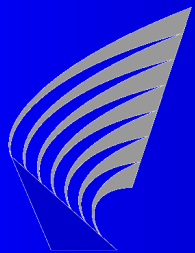


Bayesian Theory

Chapters 2.5 – 2.8



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Time Series Prediction Group

Outline

- Actions and Utilities
 - Bounds in Decision Problems
- Sequential Decision Problems
 - Complex Decision Problems
- Inference and Information
 - Reporting beliefs in Decision Problems
 - Information Theory



2.5 Actions and Utilities

- Utilities assign a numerical value for consequences through **utility function**
- Bounds or no bounds?
- Conditional expected utility
 - **Degrees of belief** for events and utilities
 - Decision criterion
- General Utility function (Def. 2.17)



2.6 Sequential Decision Problems

- Complex decision queues broken down to simpler ones
- **Backward Induction**
 - Optimal stopping problem
 - Marriage problem
 - Secretary problem
 - Optimal solution close to **Golden Ratio**
 - Needed in many real life problems



2.6.3 Design of Experiments

- Null experiment (standard experiment)
- Optimal experiment
 - Maximizing the **Unconditional Expected Utility** given the data
 - “If experiment not optimal, no experiment”
- Value of information (Def. 2.18)
- Perfect Information (Def. 2.19)



2.7 Inference and Information

- Individual “knows” something, but reports (possibly) something else
- Score function (utility function)
 - Proper (Def. 2.21), honesty
 - Quadratic (Def. 2.22), simplest
 - Local (Def. 2.23), weighting of mistakes
 - Logarithmic (Def. 2.24), KL



2.7 Information Theory

- KL Distance (Def. 2.30)
 - Loss in approximation of “true beliefs”
 - Derived from logarithmic score function
- Information from data (Def. 2.26)
- Expected info from Experiment (Def. 2.27)
 - Shannon’s Expected Information



2.8 Discussion

- Crossing to other territories
 - Information Theory
 - Probability Theory
- Questions
 - Can all problems be viewed as inference problems?
 - Is there a possibility for cyclic inference problems? Can it be solved with Bayesian?



Exercises from last week

$$x = (x_1, x_2)^T, \quad y = y$$

$$p_{x,y}(x, y) = \begin{cases} (x_1 + 3x_2)y & \text{when } x_1, x_2 \in [0, 1] \text{ and } y \in [0, 1] \\ 0 & \text{elsewhere} \end{cases}$$

$$p_y(y) = \int_0^1 \int_0^1 p_{x,y}(x, y) dx_1 dx_2 = \int_0^1 \int_0^1 (x_1 + 3x_2)y dx_1 dx_2 = 2y$$

$$p_{x_1}(x_1) = \int_0^1 p_{x,y}(x, y) dy = \int_0^1 (x_1 + 3x_2)y dy = \frac{1}{2}(x_1^2 + 3x_2^2)$$

$$p_{x_2}(x_2) = \int_0^1 p_{x,y}(x, y) dx_1 = \int_0^1 \frac{1}{2}(x_1 + 3x_2) dx_1 = \frac{3}{2}x_2 + \frac{1}{4}$$



Exercises from last week

➤ Blackjack

→ Actions?

→ Events?

→ Consequences?

→ Preference relation to actions?

➤ What it means?

