T-61.140 Signal Processing Systems

2nd mid term exam / final exam, Wed 4.5.2005 16-19, hall B.
You are NOT allowed to use any math reference book. **(Graphical) calculator allowed, erase extra memory**. Formula table with accompanying sheet. **Show clearly all steps in your results. Begin** a new problem **from a new page.**

If doing mid term exam today, you cannot do mid term exam on 16.5.2005. If doing final exam today, you cannot do final exam on 16.5.2005.

IF you do mid term exam, REPLY to 3, 4, 5, 6 and 7.
IF you do final exam, REPLY to 1, 2, 4, 5, 6 and 7.

1) (6p, final exam)

   a) What is the fundamental period N_0 of the sequence $x[n] = \cos((\pi/8)n) + \sin((\pi/5)n - \pi/8)$

   b) Is the discrete-time filter $y[n] = x[n] + 0.5x[n-1] + 0.25$ linear? Is it time-invariant? Compute or explain.

   c) The impulse response of the LTI filter is $h[n] = 2^{n-3}u[-n+3]$. Is it causal? Is it stable? Show using the definitions.

2) (6p, final exam) Consider the system in Figure 1. Two LTI systems H_1 and H_2 are unknown. A sequence $x[n]$ is known and two output sequences $w[n]$ and $y[n]$ are read as shown in the figure

$$\begin{aligned} x[n] &= \delta[n] + 2\delta[n-2] - \delta[n-3] \\ w[n] &= -2\delta[n] - \delta[n-1] - 4\delta[n-2] + \delta[n-4] \\ y[n] &= 2\delta[n-1] - \delta[n-2] + 3\delta[n-3] - 4\delta[n-4] - \delta[n-5] + \delta[n-6] \end{aligned}$$

Define the impulse responses $h_1[n]$ and $h_2[n]$ of the unknown systems and draw the block diagrams (flow diagrams) for these filters.

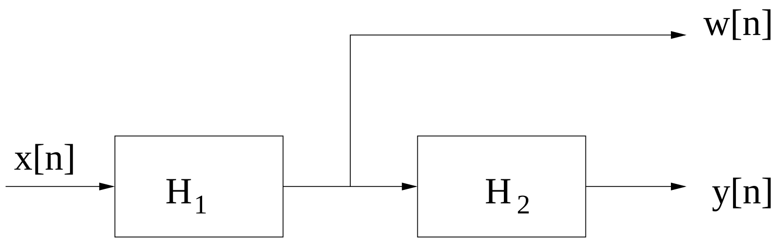

Figure 1: Sequences and filters of Problem 2.

3) (6p, mid term exam) Statements. If you think that the statement is true, write T. If you think that the statement is false, write F. Each correct answer +1 p, wrong answer −0.25 p, or no answer 0 p. You don't have to explain your results.

a) Fourier transform of the sequence $x[n] = (-0.5)^n u[n]$ is $X(e^{j\omega}) = 1/[1 - 0.5e^{-j\omega}]$.
b) The length of the impulse response of the filter $H(e^{j\omega}) = 1 - 0.1e^{-3j\omega}$ is four.
c) The filter $H(e^{j\omega}) = e^{j\omega} + 0.5e^{2j\omega} + 0.25e^{3j\omega} + 0.125e^{4j\omega} + \ldots$ has the corresponding difference equation $y[n] = x[n+1] + 0.5y[n+1]$.
d) The group delay of the filter $H(e^{j\omega}) = 1 + e^{-j\omega}$ is constant.
e) Zero-order hold applied for sequence $x[n] = \cos((\pi/2)n)$ produces a pure cosine component $x_r(t) = \cos(2\pi(0.25f_s)t)$ at sampling frequency f_s.
f) The frequency components over the sampling frequency f_s of a continuous real signal $x(t)$ can be observed after sampling as folded components at some frequencies below $f_s/2$.

4) (8p, mid term exam, final exam) The following program is reading the input sequence (`input_stream`) from A/D-converter, and does some numerical computation, and returns the output sequence back to D/A-converter (`output_stream`). Discrete-time filter is represented with pseudo code, where read / write / assign / computation operations use 16-bit numbers:

```
y0 := 0; y1 := 0; x0 := 0; x1 := 0; x2 := 0;   % init
while TRUE {
  x2 := x1; x1 := x0; y1 := y0;
  x0 := read_next_item(input_stream);
  y0 := x0  + 1.8 * x1 + 0.82 * x2 + 0.9 * y1;
  write_item(output_stream, y0);
}
```

a) Write down the difference equation of the filter and draw the flow diagram (block diagram) using the notations used in the course.
b) Define the frequency response of the filter $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$.
c) Compute the missing values of frequency response $H(e^{j\omega})$ and amplitude response $|H(e^{j\omega})|$ in Table 1. Sketch the graph of the amplitude repsonse.
d) What is the filter order?
e) What is the value of the impulse response $h[n]$ at $n = 100$?

ω	$H(e^{j\omega})$	$\lvert H(e^{j\omega})\rvert$
0		
$\pi/2$		
π		

Table 1: Problem 4, frequency and amplitude response of the filter.

5) (4p, mid term exam, final exam) See Figure 2. There is a continuous signal $x(t)$ with a solid line, and a discrete-time signal $x[n]$, which is sampled from $x(t)$, with circles.

a) Read from the figure, what is the sampling frequency f_s.

b) The spectrum analysis of the continuous signal gives two peaks at frequencies 80 and 220 Hz, and some weak white noise. What do you know now about the sequence $x[n]$? Explain.

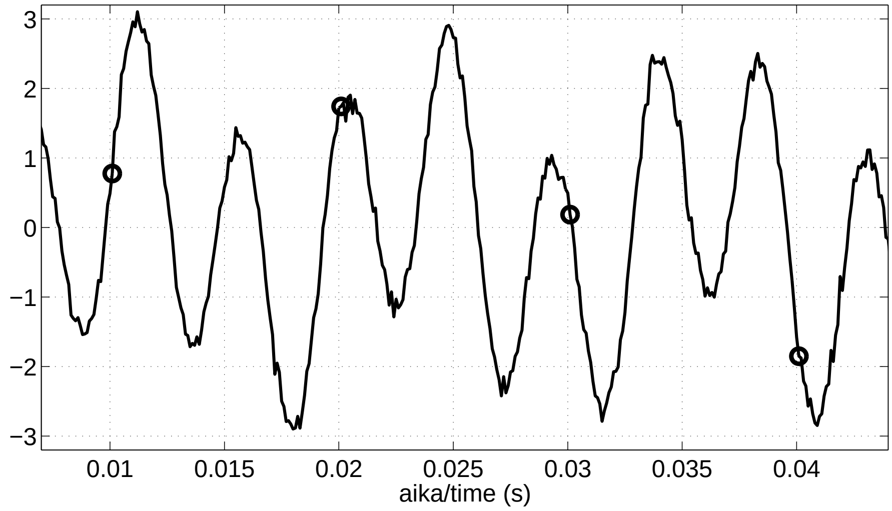

Figure 2: Problem 5: signal $x(t)$ with solid line and sequence $x[n]$ with circles.

6) (6p, mid term exam, final exam) Consider two LTI filters that are shown in Figure 3.

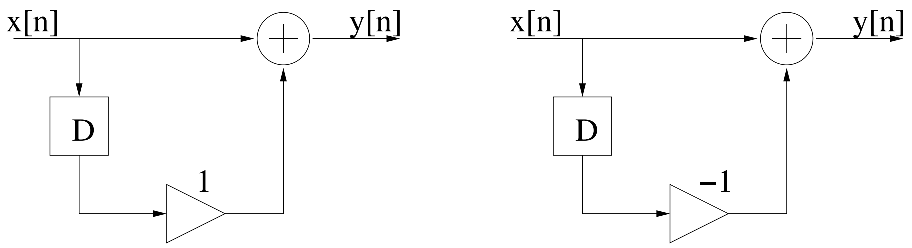

Figure 3: Problem 6, two LTI filters H_1 and H_2.

a) Compute the impulse response of the cascade system $h_c[n] = h_1[n] * h_2[n]$.

b) Determine the frequency response $H_c(e^{j\omega})$ of the cascade filter.

c) Compute values of amplitude response $|H_c(e^{j\omega})|$ and sketch graph of amplitude response. What is the type of the filter: lowpass / highpass / bandpass / bandstop / all-pass?

d) The phase response of the filter is $\angle H_c(e^{j\omega}) = -\omega + \pi/2$. Is the phase response linear? What is the group delay $\tau_c(\omega) = -\frac{\mathrm{d}}{\mathrm{d}\omega}\angle H_c(e^{j\omega})$ of the filter?

7) (2p mid term exam, 1p final exam)

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