T-61.140 Signal Processing Systems

2nd mid term exam / final exam, Thu 8th May 2003 16-19 hall M
No mathematical reference book. Formulae given - use them! A graphical calculator allowed (extra memory must be cleared).
2nd mid term exam: Write on top "MID TERM EXAM" and **reply to problems 3, 4, 5 and 6.**
Final exam: Write on top "FINAL EXAM" and **reply to problems 1, 2, 4, 5 and 6.**
ATT! If you are doing now the 2nd MTE, you cannot do that on 15.5.2003. If you are doing now the final exam, you cannot do that on 15.5.2003.

1) (Final exam, 6p) Properties of systems and signals. Calculate or explain clearly.

   a) A discrete system is defined with a difference equation $y[n] = -x[n+1]+2x[n]-x[n-1]$. Is the system linear? Is it time-invariant?

   b) A LTI system is defined with an impulse response $h[n] = \left(\frac{1}{3}\right)^n u[n+1]$. Is the system stable? Is it causal?

   c) Let us know a discrete sequence $x[n] = \cos(\frac{\pi}{6}n) + 2\,\sin(\frac{\pi}{9}n + \frac{\pi}{4})$. Is $x[n]$ periodic? If it is, what is the fundamental period N_0?

2) (Final exam, 6p) Let us examine discrete-time filters defined with difference equations

$$\begin{aligned} y_1[n] &= x[n] + 2x[n-1] + 3x[n-2] \\ y_2[n] &= -x[n] - 2x[n-2] \end{aligned}$$

   a) Draw the block (flow) diagrams for both cases.

   b) Compute the impulse response $h_p[n]$ of the parallel system.

   c) Compute the impulse response $h_c[n]$ of the cascade (series) system.

   d) What is the output $y[n]$ of the cascade system, if the input is $x[n] = \delta[n] + 10\delta[n-10] + 100\delta[n-100] + 1000\delta[n-1000]$.

3) (Mid term exam, 3 x 2p = 6p) Reply to **three** statements, if they are TRUE (T) or FALSE (F). Explain briefly but unambiguously.

   a) Any sequence $x[n]$ can be recovered from its amplitude spectrum $|X(e^{j\omega})|$, if there are enough computational power.

   b) The frequency response $H(e^{j\omega}) = e^{-j2\omega}$ attenuates (does not amplify) high frequencies.

   c) The LTI system with the impulse response $h[n] = \delta[n+2] + \delta[n]$ is linear-phase.

   d) If $H_l(e^{j\omega})$ is an **ideal** lowpass filter, then $H(e^{j\omega}) = 1 - H_l(e^{j\omega})$ is an ideal highpass filter.

4) (Final exam/Mid term exam, 6p) The impulse response of a LTI system is

$$h[n] = 0.9^n u[n] + (-0.9)^n u[n]$$

   a) What is the frequency response $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$.

   b) Is the filter FIR or IIR? Is the algorithm recursive or not? What is the order of the filter?

   c) Sketch the amplitude response $|H(e^{j\omega})|$. Is the filter of type lowpass, highpass, bandpass, bandstop or an allpass filter?

   d) Draw the block (flow) diagram of the filter.

5) (Final exam/Mid term exam, 6p) Consider a continuous-time signal $x(t)$ which consists of two cosine components:

$$x(t) = A_1 \cos(2\pi f_1 t + \theta_1) + A_2 \cos(2\pi f_2 t + \theta_2)$$

In Figure 1 there are the components and the sum signal in bottom. The scale in x-axis is about $0 \ldots 0.1$ seconds.

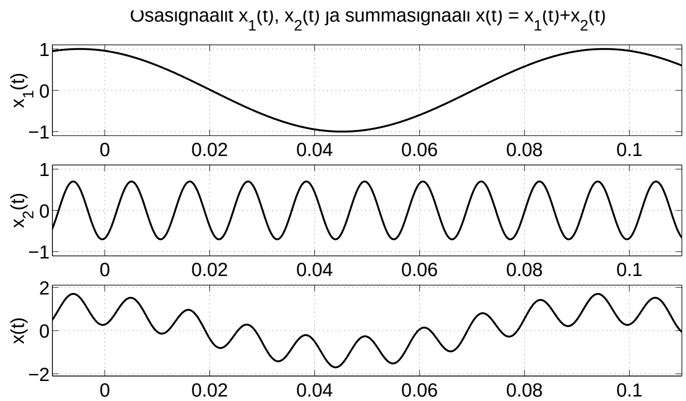


Figure 1: Two cosine signals and the sum of them.

a) Sketch the amplitude spectrum $|X(j\omega)|$ of the signal $x(t)$.

b) What is the smallest sampling frequency, with which there is no aliasing in $x(t)$?

c) Take samples with $f_s = 100$ Hz. Sketch the spectrum $|X(e^{j\omega})|$ of the sequence $x[n]$ in range $0 \ldots 50$ Hz.

d) Sketch the waveform of the reconstructed signal $x_r(t)$ in the time domain.

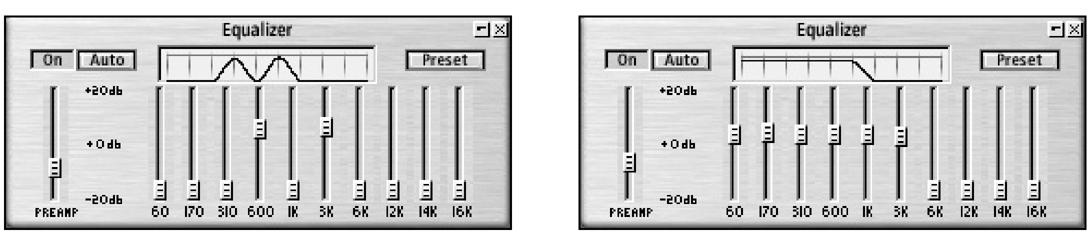
6) (Final exam/Mid term exam, 6p) **Reply to either A or B.**

6A) The Fibonacci sequence can be represented in the form of $y[n] = \{\underline{1}, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots\}$.

a) Generate the sequence using the second-order recursive LTI system. Assign initial values of the registers $y[0]$ and $y[1]$ correctly. (Keep the input in zero, $x[n] \equiv 0$, for all n.) Draw the block (flow) diagram of the recursive implementation.

b) How is it possible to compute the value at the moment n non-recursively? Hint: What is the impulse response of the system in a)? Compute $y[2003]$.
(Vinkki pyörittelyyn: $x_1 = 3.14^{2003} = 10^{2003 \cdot \log_{10} 3.14}$,
$x_2 = 10^{3.14} = 10^3 \cdot 10^{0.14} \approx 1.38 \cdot 10^3$, $\quad x_3 = \log_{10} 3.14 = (\ln 3.14)/(\ln 10)$.)

6B) Consider a simple equalizer which can be found in several software programs, like WinAmp in Figure 2. The frequency area is divided to channels (WinAmp: 60, 170, 310, 600, 1k, 3k, 6k, 12k, 14k, 16k) and it is assumed that the voice is sampled with 44100 Hz. How do you use equalizer and how does it affect voice? Explain, how an equalizer could be constructed using the knowledge from the course?



(a) (b)

Figure 2: Two examples on WinAmp equalizers.