T-61.246 Digital Signal Processing and Filtering

Rehearsal exam 3.12.2002, 5.12.2002.

2nd MTE Wed 11.12.2002 at 9-12. Halls A, B and C.
Final exam Fri 13.12.2002 klo 12-15. Halls M and K.
If you are doing the 2nd MTE, reply to 4, 5, 6.
If you are doing the final exam, reply to 1, 2, 3, 5, 6.

Some other possible problems: convolutio (exam), properties of discrete-time systems (exam), multirate, FIR filter design, statements.

1. (3p, EXAM) Are the following sequences periodic, and if they are, what is the length of the fundamental period?

   a) $x_a[n] = \mu[n] + \mu[-1-n]$
   b) $x_b[n] = \sum_{k=-\infty}^{\infty}(\delta[n-3k] - \delta[n+1-2k])$
   c) $x_c[n] = 2\sin(\frac{2\pi 3}{7}n) + 3\cos(\frac{\pi}{9}n)$

2. (3p, EXAM) In Figure 1 there's a spectrum $|X(j\omega)|$ of a continuous-time signal $x(t)$. One takes 100 samples from the signal for DSP. The sequence is filtered using an ideal lowpass filter, whose cut-off frequency is 1/4 of the sampling frequency, $\omega_c = \omega_s/4$. After that the sequence is reconstructed to continuous signal. Which frequencies does it then exist?

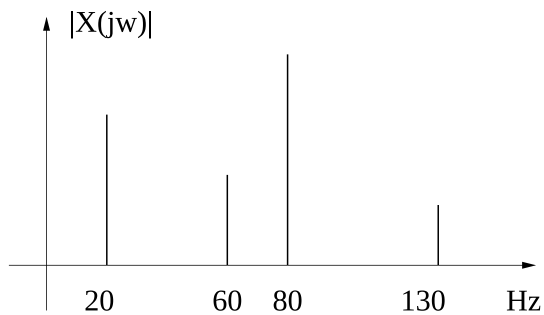


Figure 1: The spectrum of the signal $x(t)$ in Problem 2.

3. (6p, EXAM) Consider a block diagram of Figure 2.

   a) What is the difference equation or group of equations that express the computation.
   b) What is the transfer function $H(z)$?
   c) Draw the pole-zero-diagram and sketch the apmlitude response $|H(e^{j\omega})|$.
   d) What is the impulse response $h[n]$?

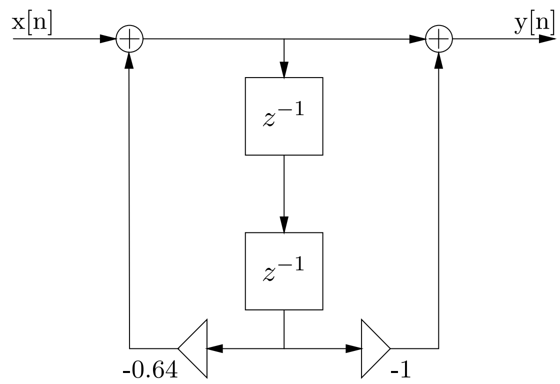
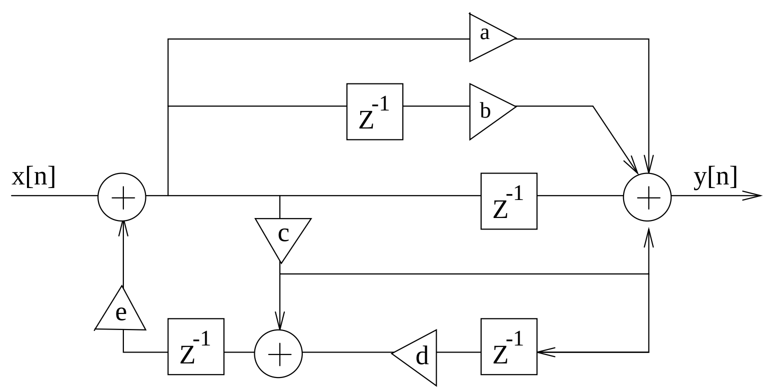


Figure 2: The block diagram of Problem 3.

4. (6p, MTE2) Convert the filter structure shown below to a canonic form so that the transfer function remains the same.



5. (6p, MTE2, EXAM) Design and compute a digital first-order Butterworth low-pass filter, whose cut-off frequency is $f_c = 3300$ Hz and the sampling frequency $f_s = 22050$ Hz. Use the bilinear transform. The transfer function of the analog Butterworth filter with cut-off frequency $\Omega_c = 2\pi f_c$ is:

$$H(s) = \frac{\Omega_c}{s + \Omega_c}$$

6. (6p, MTE2, EXAM) Consider a first-order recursive filter, whose transfer function is

$$H(z) = \frac{1}{1 - az^{-1}}, \qquad \text{where } a = 0.5$$

   a) Is the filter stable?
   b) Define the difference equation for input/output relation and draw the block diagram.
   c) Examine the impulse response of the filter using the precision of four bits (sign bit + 3 bits) and rounding arithmetics. Suppose that all absolute values are scaled smaller than one. For example, $0_\Delta 111 \triangleq 7/8$ (biggest positive number), $a = 0.5 \triangleq 0_\Delta 100$. How does the filter behave? Why?