T-61.3010 Digital Signal Processing and Filtering

1st mid term exam, Sat 11th March 2006 at 10-13. Halls A-M.
You are allowed to do 1st MTE only once either 7.3. or 11.3.

You are not allowed to use any calculators or math tables. A table of formulas is delivered as well as a form for Problem 1.

All papers have to be returned. **In Problem 1 you must return a specific paper.** Problem statement and a paper of formulas you can keep.

Start a new task from a **new page**. Write all **intermediate steps**.

1) (14 x 1p, max 12 p) Multichoice. There are 1-4 correct answers in the statements, but choose **one and only one.** Fill your solutions in a specific **form** by painting the whole square.

   Correct answer +1 p, wrong answer −0.5 p, no answer 0 p. You do not need write why you chose your option. Reply to as many as you want. The maximum number of points is 12, and the minimum 0.

   1.1 In Matlab assignments human speech was analyzed, specially a short part of a vowel /i/, which is here denoted as $x(t)$.
   (A) It was found out that $x(t)$ is practically periodic $x(t) \approx x(t+T)$
   (B) It was found out that $x(t)$ is mathematically periodic $x(t) = x(t+T)$
   (C) Signal $x(t)$ produces a spectrum, where there are peaks at "fundamental frequency" (about 200 Hz) and its harmonics
   (D) Signal $x(t)$ produces a spectrum, which is looking like a triangular

   1.2 Two point moving averaging filter:
   (A) impulse response is $h[n] = 0.5\delta[n] - 0.5\delta[n-1]$
   (B) transfer function $H(z) = 0.5(1 + z^{-1})$
   (C) amplifies quick changes in the signal
   (D) difference equation is $y[n] = \frac{x[n]+x[n-1]}{2}$

   1.3 Impulse response of filter is $h[n] = (-0.5)^{n-2}\mu[-n+2]$:
   (A) filter is stable
   (B) filter is FIR
   (C) filter is not causal
   (D) region of convergence (ROC) of transfer function $H(z)$ of the filter is $|z| < 0.5$

   1.4 Consider a sequence $x[n] = \cos((\pi/8)n) - \sin((\pi/4)n) + 2\cos((\pi/3)n - \pi/4)$. What can be said about period of sequence $x[n]$?
   (A) There is no fundamental period N_0
   (B) Fundamental period is $N_0 = 96$
   (C) Fundamental period is $N_0 = 48$
   (D) Fundamental period is $N_0 = 16$

   1.5 Consider signal $x(t) = x_1(t) + x_2(t) + x_3(t)$, where fundamental periods of each subsignal are $T_1 = 8$, $T_2 = 10$, and $T_3 = 20$. What can be said about period of sequence $x(t)$?
   (A) Fundamental period T_0 depends on sampling frequency
   (B) Fundamental period $T_0 = 4$
   (C) Fundamental frequencies f_1, f_2 and f_3 of subsignals can be expressed as multiples of f_0, which is the fundamental frequency of $x(t)$
   (D) Signal is periodic with period $T = 100$

1.6 The impulse response $h[n]$ of filter in Figure 2(a) convolved with input sequence $x[n] = 0.5\delta[n] - 0.5\delta[n-1]$

(A) produces a finite length output sequence

(B) cannot be computed because filter is not causal

(C) unit step response goes to zero at $n = 1$

(D) sum $\sum_{n=-\infty}^{\infty} |y[n]|$, where $y[n]$ is the output sequence, converges and is finite

1.7 Consider a linear convolution $y[n] = h[n] \circledast x[n]$. Define $w[n]$ as a new convolution: $w[n] = h[n - N_1] \circledast x[n - N_2]$.

(A) $w[n] = y[n]$

(B) $w[n] = (N_1 \cdot N_2)\, y[n - (N_1 + N_2 - 1)]$

(C) $W(e^{j\omega}) = e^{j(N_1+N_2)\omega}\, Y(e^{j\omega})$

(D) $W(e^{j\omega}) = e^{j(-N_1-N_2)\omega}\, Y(e^{j\omega})$

1.8 Consider a filter $H(z) = 1 - 0.5z^{-8}$.

(A) The amplitude response is in Figure 1(a)

(B) The pole-zero diagram is in Figure 1(d)

(C) Filter is second-order FIR

(D) Length of impulse response $h[n]$ is eigth

1.9 Transfer function $H(z) = [1 - 0.3z^{-1} + 0.2z^{-2}]/[1 + 0.9z^{-2}]$.

(A) Phase response of the filter is nonlinear

(B) Flow/Block diagram is in Figure 2(b)

(C) Impulse response is $h[n] = 0.9^n\mu[n] - 0.3 \cdot 0.9^{n-1}\mu[n-1] + 0.2 \cdot 0.9^{n-2}\mu[n-2]$

(D) Magnitude response is in Figure 1(b)

1.10 Consider a real sequence $x[n]$

(A) Discrete-time Fourier transform of $x[n]$ is periodic every π

(B) Absolute value of discrete-time Fourier transform of $x[n]$ is an odd function

(C) Angle/phase of discrete-time Fourier transform of $x[n]$ is an even function

(D) Discrete-time Fourier transform of $x[n]$ can be real-valued

1.11 Consider an inverse transform $h[n]$ of filter $H(z) = [1 - 0.2z^{-1}]/[1 + 0.6z^{-1} + 0.05z^{-2}]$, with region of convergence (ROC) $|z| > 0.5$. What is $h[n]$?

(A) $h[n] = 0.6^n\mu[n] - 0.2 \cdot 0.05^{n-1}\mu[n-1]$

(B) $h[n] = 1.75 \cdot (-0.5)^n\mu[n] - 0.75 \cdot (-0.1)^n\mu[n]$

(C) $h[n] = 0.5 \cdot (-0.3 + 0.2j)^n\mu[n] + 0.5z^{-1} \cdot (-0.3 - 0.2j)^{n-1}\mu[n-1]$

(D) $h[n] = 1.25 \cdot 0.5^n\mu[n] - 0.25 \cdot 0.1^n\mu[n]$

1.12 Pole-zero plot corresponding amplitude response in Figure 1(c)

(A) is in Figure 1(e)

(B) contains a pole at $z = 1$

(C) contains a zero at $z = -1$

(D) contains a zero at $\omega = \pi/2$

1.13 Consider a LTI system with impulse response $h[n] = (-1)^{n-2}\mu[n+2]$ and input $x[n] = \delta[n+4] - 3\delta[n+3] + 2\delta[n+2]$. Output $y[n] = h[n] \circledast x[n]$ is computed.

(A) $y[2006] = -6$

(B) $y[2006] = 0$

(C) $y[2006] = 6$

(D) $y[2006] = \delta[n - 2002] - 3\delta[n - 2003] + 2\delta[n - 2004]$

1.14 In the parallel connection of two LTI systems h_1 and h_2

(A) the pole-zero plot of the whole system h is received by computing poles and zeros from both subsystems and drawing them into the same pole-zero-plot.

(B) impulse response of the whole system h is derived by summing impulse responses of subsystems

(C) impulse response of the whole system h is derived by multiplication of impulse responses of subsystems

(D) impulse response of the whole system h is derived by convolving impulse responses of subsystems

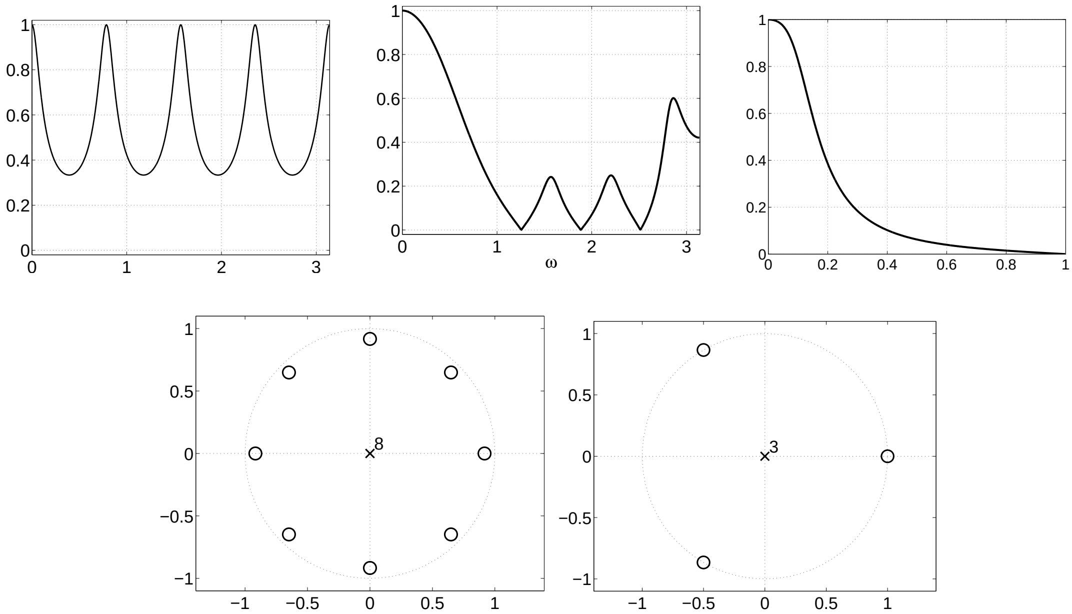

Figure 1: Figures for multichoice problem, ylärivi (a), (b), (c), alarivi: (d), (e).

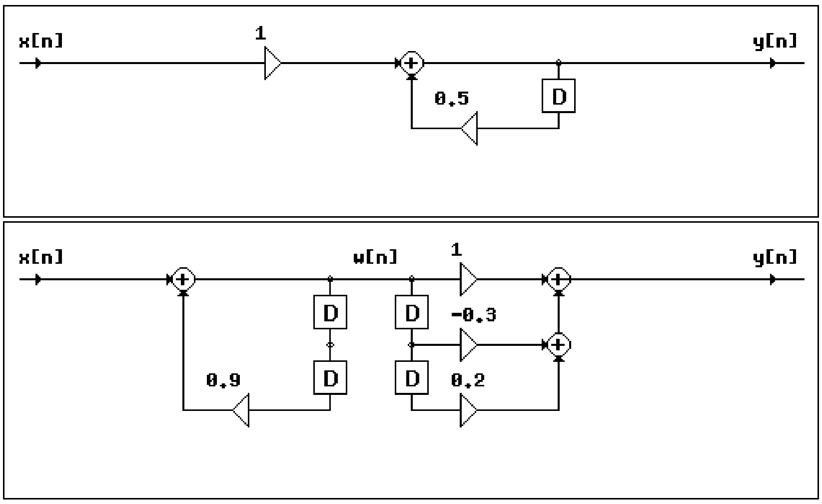

Figure 2: Figures for multichoice problem, (a), (b).

2) (6 p) Analog signal $x(t) = \sum_i A_i \cos(2\pi f_i t + \theta_i)$ consists of five frequency components { $f_1 = 400$ Hz, $A_1 = 2$, $\theta_1 = 0.6$ }, { $f_2 = 600$ Hz, $A_2 = 7$, $\theta_2 = 0.1$ }, { $f_3 = 5400$ Hz, $A_3 = 3$, $\theta_3 = 0.3$ }, { $f_4 = 9200$ Hz, $A_4 = 10$, $\theta_4 = 0.01$ } ja { $f_5 = 10200$ Hz, $A_5 = 5$, $\theta_5 = 0.0$ }.

a) Signal is periodic. What is the fundamental frequency f_0?

b) Sketch the spectrum $|X(j\Omega)|$ of signal $x(t)$ in range $f \in [0 \dots 20]$ kHz.

c) Signal is sampled with sampling frequency $f_s = 10$ kHz. Sketch the spectrum $|X(e^{j\omega})|$ of sampled sequence $x[n]$.

d) Sequence $x[n]$ is filtered with a filter, whose pole-zero plot is in Figure 3. After that, the filtered sequence $y[n]$ is reconstructed (ideally) to continuous-time $y_r(t)$. Sketch the spectrum $|Y_r(j\Omega)|$ in range $f \in [0 \dots 20]$ kHz.

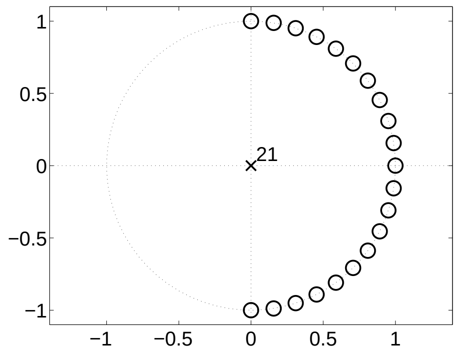

Figure 3: Pole-zero plot of the filter.

3) (6 p) Consider a stable and causal discrete-time LTI system S_1, whose zeros z_i and poles p_i are at

$$\text{zeros:} \quad z_1 = 1, \quad z_2 = 1$$
$$\text{poles:} \quad p_1 = 0.18$$

Add a LTI FIR filter S_2 in parallel with S_1 as shown in Figure 4 so that the whole system S is causal second-order bandstop filter, whose minimum is approximately at $\omega \approx \pi/2$ and whose maximum is scaled to one. What are transfer functions S_2 and S? Show clear intermediate steps.

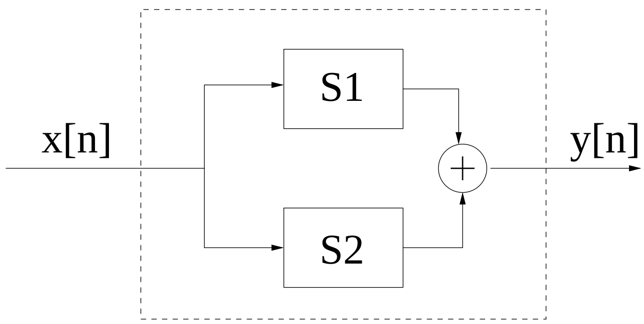

Figure 4: Filter S constructed from LTI subsystems S_1 and S_2.