T-61.3010 Digital Signal Processing and Filtering

Mid term exam 1, Sat 7.3.2009 at 10-13, main building.

You are allowed to do MTE1 only once either 7.3. or 13.3.

You are not allowed to use any calculators or math reference books. A list of formulas is delivered in the exam. A special form is delivered for Problem 1.

Return a special form and the other answer paper separately. Both ones have to have at least student number and name written on. Problem paper and the formulas you may keep.

Problem 3 is a course feedback which is open from Sat 7-March to Mon 23-March 2009.

1) (0-12 p) Multichoice statements. There are 1-4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 12 and the minimum 0.

1.1 Consider a sequence $x[n] = x_1[n] + x_2[n] + x_3[n]$, where fundamental periods of subsequences are $N_1 = 8$, $N_2 = 10$ and $N_3 = 20$. What can be said about period of sequence $x[n]$?
- **(A)** There is no fundamental period N_0
- **(B)** Fundamental period is $N_0 = 2$
- **(C)** Fundamental normalized angular frequency is $\omega_0 = 2\pi/N_0 = \pi/20$
- **(D)** Sequence is periodic with period $N = 8 \cdot 10 \cdot 20 = 1600$

1.2 Compute linear convolution $y[n] = h[n] \circledast x[n]$ of sequences $x[n] = \delta[n+2] + \delta[n+1] + \delta[n] = \{1,\ 1,\ \underline{1}\}$ and $h[n] = \delta[n+1] - \delta[n] = \{1,\ \underline{-1}\}$ where underline shows the origin.
- **(A)** Length of $y[n]$ is 5
- **(B)** $y[n] = 0$, when $n < 0$
- **(C)** $y[0] = -1$
- **(D)** $y[0] = 1$

1.3 Compute deconvolution, when $y[n] = h[n] \circledast x[n]$, and we have $x[n] = \{1,\ \underline{-2},\ 1\}$ and $y[n] = \{\underline{-1},\ 1,\ 2,\ -3,\ 1\}$ where underline shows the origin. Hence, the unknown $h[n]$ is of form
- **(A)** $h[n] = a \cdot \delta[n+1] + b \cdot \delta[n] + c \cdot \delta[n-1] + d \cdot \delta[n-2]$
- **(B)** $h[n] = b \cdot \delta[n] + c \cdot \delta[n-1] + d \cdot \delta[n-2]$
- **(C)** $h[n] = b \cdot \delta[n] + c \cdot \delta[n-1] + d \cdot \delta[n-2] + e \cdot \delta[n-3]$
- **(D)** $h[n] = c \cdot \delta[n-1] + d \cdot \delta[n-2] + e \cdot \delta[n-3]$

where $\{a,\ b,\ c,\ d,\ e\} \in \mathbb{R}$ and non-zero.

1.4 What is the difference equation corresponding a LTI system shown in Figure 1?
- **(A)** $y[n] - 0.9y[n-1] + 0.7y[n-2] = 0.5x[n] + 0.5x[n-2]$
- **(B)** $y[n] + 0.9y[n-1] - 0.7y[n-2] = 0.5x[n] + 0.5x[n-2]$
- **(C)** $y[n] = 0.5x[n] + 0.5x[n-2] + 0.45x[n-1] - 0.35x[n-2]$
- **(D)** None of above is true

1.5 Consider a stable and causal LTI filter whose poles are $p_1 = -0.8$, $p_2 = -0.5$, and $p_3 = 0.5$, and zeros $z_1 = -1$, $z_2 = 0.8$, and $z_3 = 1$.
- **(A)** Impulse response $h[n]$ of the filter is symmetric
- **(B)** Difference equation of the filter is $y[n] = K \cdot \big(x[n] - x[n-1] + 0.8x[n-2] + x[n-3] + 0.8y[n-1] + 0.5y[n-2] - 0.5y[n-3]\big)$, where K is a scaling factor
- **(C)** Transfer function of the filter is $H(z) = K \cdot \frac{1-0.8z^{-1}-z^{-2}+0.8z^{-3}}{1+0.8z^{-1}-0.25z^{-2}-0.2z^{-3}}$, $|z| > 0.8$, where K is a scaling factor
- **(D)** Magnitude response of the filter is $|H(e^{j\omega})| = 1$ for all frequencies $\omega \in [0,\ \pi]$

1.6 Difference equation of a LTI system is $y[n] = x[n] + 0.4x[n-1] - 0.21x[n-2] + 1.8y[n-1] - 0.82y[n-2]$. Magnitude response $|H(e^{j\omega})|$ scaled between $0 \ldots 1$ is
- **(A)** in Figure 2(a)
- **(B)** in Figure 2(b)
- **(C)** in Figure 2(c)
- **(D)** in Figure 2(d)

1.7 Assume that z-transform is known for sequences $g[n]$ and $h[n]$. What does the following lines especially prove for us?

$$\begin{aligned}
&\sum_{n=-\infty}^{+\infty} \Big(\sum_{k=-\infty}^{+\infty} g[k]h[n-k] \Big) z^{-n} \\
=\ &\sum_{k=-\infty}^{+\infty} g[k] \Big(\sum_{n=-\infty}^{+\infty} h[n-k] z^{-n} \Big) \\
=\ &\sum_{k=-\infty}^{+\infty} g[k] \Big(\sum_{m=-\infty}^{+\infty} h[m] z^{-(m+k)} \Big) \quad , \ m = n-k \\
=\ &\Big(\sum_{k=-\infty}^{+\infty} g[k] z^{-k} \Big) \Big(\sum_{m=-\infty}^{+\infty} h[m] z^{-m} \Big)
\end{aligned}$$

- **(A)** Linear convolution is commutative operation
- **(B)** Product of sequences of infinite length is also infinitely long
- **(C)** z-transform of a convolution of sequences g and h is the same as product of their z-transforms
- **(D)** z-transform of a product of sequences g and h is the same as convolution of their z-transforms

1.8 Which of the following discrete-time sequences is both linear and time-invariant (LTI)?
- **(A)** Filter in Figure 3(a)
- **(B)** Median filter, where $y[n]$ is median of samples $\{x[n], x[n-1], \ldots, x[n-L+1]\}$, where L is number of input samples ("moving median filter")
- **(C)** $y[n] = x[n] + 1$
- **(D)** $y[n] = x_1[n] \cdot x_2[n]$

1.9 Speech is recorded into a computer with the sampling frequency of $f_T = 10000$ Hz so that the length of the sequence is 29001.
- **(A)** According to sampling theorem we can observe frequences up to 9999 Hz
- **(B)** According to sampling theorem we can observe only frequencies whose period is smaller that 0.2 milliseconds
- **(C)** Length of the speech in seconds cannot be determined with given information
- **(D)** None of above is true

1.10 Examine a stable and causal filter

$$H(z) = \frac{1 - z^{-2}}{1 - 1.5z^{-1} - 0.56z^{-2}}$$

- **(A)** Phase response $\angle H(e^{j\omega})$ of the filter is linear
- **(B)** Phase response of the filter is in Figure 3(b)
- **(C)** Pole-zero-plot is in Figure 3(c)
- **(D)** Group delay $\tau(\omega)$ of the filter is not constant with respect to ω

1.11 Consider sequences $h[n] = 0.8^n \mu[n]$ and $x[n] = 3 \cdot \sum_{k=0}^{\infty} (-0.5)^{(k/42)} \delta[n - (k/42)]$, and their linear convolution $y[n] = h[n] \circledast x[n]$. We know that
- **(A)** the sum of sequences $y[n]$ is $S = \sum_{n=\infty}^{\infty} y[n] = 10$
- **(B)** the sum of sequences $y[n]$ is $S = \sum_{n=\infty}^{\infty} y[n] = 30$
- **(C)** the sum of sequences $y[n]$ is $S = \sum_{n=\infty}^{\infty} y[n] = 42$
- **(D)** the sum of sequences $y[n]$ is $S = \sum_{n=\infty}^{\infty} y[n]$ does not converge

Notice that $\delta[n] = 0$, if $n \notin \mathbb{Z}$

1.12 The impulse response of the filter is $h[n] = (-0.8)^n \mu[n]$ and the output $y[n] = 2 \cdot (-0.8)^n \mu[n] - (-0.4)^n \mu[n]$.
- **(A)** The filter is averaging the input
- **(B)** The filter is both unstable and non-causal
- **(C)** The input is $x[n] = 2\delta[n] - (0.5)^n \mu[n]$
- **(D)** The input is $x[n] = (-0.4)^n \mu[n]$

2) (6 p) In the first part of this course we have examined inputs $x[n]$, digital LTI systems $h[n]$ which process them and outputs $y[n]$. These can be processed both in time- and frequency-domain.

Write down an essay about "filtering signals with digital LTI filters".

3) (1 p) Course feedback. Questionnaire `http://www.cis.hut.fi/Opinnot/T-61.3010/VK1_K2009/kyselyVK1_en.shtml` is open till 23-Mar 2009.

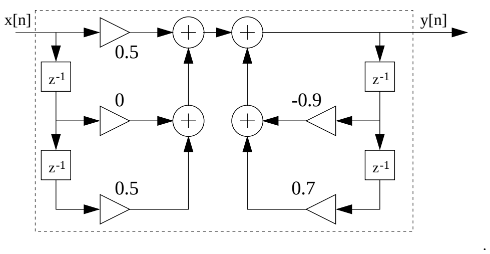

Figure 1: Problem 1.4: block diagram of a filter.

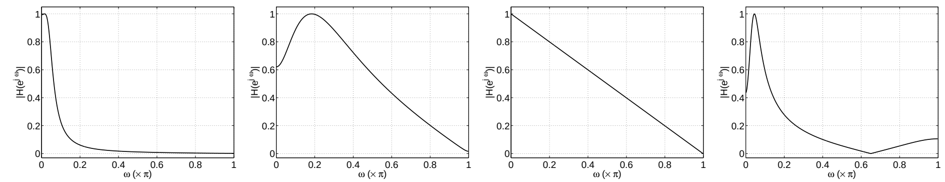

Figure 2: Problem 1.6: Magnitude response, options **(A)** , **(B)** , **(C)** , **(D)** .

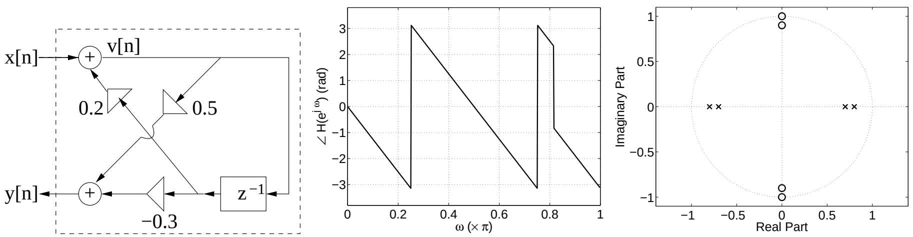

Figure 3: Problems 1.8 and 1.10: (a) 1.8 **(A)** , (b) 1.10 **(B)** , (c) 1.10 **(C)** .