# T-61.3010 Digital Signal Processing and Filtering

Mid term exam 1, Fri 13.3.2009 at 9-12, hall A.

**You are allowed to do MTE1 only once either 7.3. or 13.3.**

You are not allowed to use any calculators or math reference books. A list of formulas is delivered in the exam. A special form is delivered for Problem 1.

Return a special form and the other answer paper separately. Both ones have to have at least student number and name written on. Problem paper and the formulas you may keep.

Problem 3 is a course feedback which is open from Sat 7-March to Mon 23-March 2009.

1) (0-12 p) Multichoice statements. There are 1-4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 12 and the minimum 0.

1.1 Consider a sequence $x[n] = x_1[n] + x_2[n] + x_3[n]$, where fundamental periods of subsequences are $N_1 = 4$, $N_2 = 8$ and $N_3 = 20$. What can be said about period of sequence $x[n]$?
**(A)** There is no fundamental period $N_0$
**(B)** Fundamental period is $N_0 = 20$
**(C)** Fundamental normalized angular frequency is $\omega_0 = 2\pi/N_0 = \pi/20$
**(D)** Sequence is periodic with period $N = 4 \cdot 8 \cdot 20 = 640$

1.2 Compute linear convolution $y[n] = h[n] \circledast x[n]$ of sequences $x[n] = \delta[n+1] + \delta[n] + 2\delta[n-1] = \{1,\ \underline{1},\ 2\}$ and $h[n] = \delta[n+1] - \delta[n] = \{1,\ \underline{-1}\}$ where underline shows the origin.
**(A)** $y[n] = 0$, when $n < 0$
**(B)** Left-most non-zero value of sequence $y[n]$ is at $n = 1$
**(C)** $y[0] = -1$
**(D)** $y[0] = 1$

1.3 Compute deconvolution, when $y[n] = h[n] \circledast x[n]$, and we have an impulse response $h[n] = \{1,\ \underline{-2}\}$ and $y[n] = \{\underline{2},\ 0,\ 0,\ 0,\ -d\}$, where underline shows the origin. Hence, the input $x[n]$ is of form
**(A)** $x[n] = 2 \cdot \delta[n+1] - d \cdot \delta[n-3]$
**(B)** $x[n] = 2 \cdot \delta[n+1] + a \cdot \delta[n] + b \cdot \delta[n-1] + c \cdot \delta[n-2] + d \cdot \delta[n-3]$
**(C)** $x[n] = 2 \cdot \delta[n-1] + c \cdot \delta[n-5]$
**(D)** $x[n] = 2 \cdot \delta[n-1] + a \cdot \delta[n-2] + b \cdot \delta[n-3] + c \cdot \delta[n-4]$
where $\{a,\ b,\ c,\ d\} \in \mathbb{R}$ and non-zero.

1.4 LTI system is defined by the difference equation $y[n] - 0.1y[n-1] + 0.2y[n-2] = 0.5x[n] + 0.5x[n-2]$
**(A)** Corresponding block diagram is in Figure 1(a)
**(B)** Corresponding block diagram is in Figure 1(b)
**(C)** Filter is of FIR type
**(D)** Filter is not causal

1.5 Magnitude response of the filter is

$$|H(e^{j\omega})| = K \cdot \frac{|1 - 0.3\,e^{-j\omega}| \cdot |1 + e^{-j\omega}|}{|1 - 0.9j\,e^{-j\omega}| \cdot |1 + 0.9j\,e^{-j\omega}|}$$

**(A)** $|H(e^{j\omega})| = 0$, when $\omega = 0$
**(B)** $|H(e^{j\omega})| \to \infty$, when $\omega \to 0.9$
**(C)** Impulse response $h[n]$ is symmetric
**(D)** If the sampling frequency is $f_T = 44100$ Hz, then the maximum amplification is reached at the frequency $f \approx 11$ kHz

1.6 LTI system is defined by the difference equation $y[n] = x[n] - 1.8x[n-1] + 0.82x[n-2] + 0.2y[n-1] + 0.15y[n-2]$. Magnitude response $|H(e^{j\omega})|$ scaled between $0 \ldots 1$ is
**(A)** in Figure 2(a)
**(B)** in Figure 2(b)
**(C)** in Figure 2(c)
**(D)** in Figure 2(d)



1.7 Consider a stable digital sixth order LTI filter, whose coefficients of impulse response $h[n]$ are real-valued.
**(A)** Frequency response $H(e^{j\omega})$ is periodic with $6\pi$: $H(e^{j(\omega_0)}) = H(e^{j(6\pi+\omega_0)})$ for all $\omega_0$
**(B)** Phase response $\angle H(e^{j\omega})$ is symmetric around y-axis: $\angle H(e^{j(-\omega_0)}) = \angle H(e^{j(+\omega_0)})$ for all $\omega_0$
**(C)** Group delay $\tau(\omega) = 6$ for all $\omega_0$
**(D)** all six poles in the pole-zero-plot are always on the unit circle

1.8 See the spectrum $|X(j\Omega)|$ of a continuous-time real signal in top row of Figure 3. Sample the signal with sampling frequency $f_s = 10$ kHz. The spectrum of the sequence $x[n]$ in bottom row of Figure 3 is
**(A)** (a)
**(B)** (b)
**(C)** (c)
**(D)** (d)

1.9 There is a screenshot from Audacity software in Figure 4. An audio signal is analysed at the interval 20.40 ... 21.30 s. The sampling frequency is $f_T = 22050$ Hz. In addition there is a spectrum estimate of the chosen signal part of 50 ms.
**(A)** It is said “I love DSP” in the audio sample
**(B)** The audio sample is an example of broadband noise
**(C)** The chosen audio part is not periodic in the strict mathematical sense ($x(t) \equiv x(t+T)$) but “almost periodic” (“quasi-periodic”)
**(D)** If it is known that the audio is originated from a piano, eight neighbour keys are pressed at the same as shown in the spectrum

1.10 Assume a sequence $x[n] = \delta[n-K] + \delta[n-K-1]$, where $K \in \mathbb{Z}_+$. Compute a linear convolution

$$y[n] = x[n] \circledast (x[n] \circledast (x[n] \circledast (x[n] \circledast (x[n] \circledast x[n]))))$$

**(A)** The length of the sequence $y[n]$ is $L_y(K) = 6K - 5$
**(B)** $y[n] = 0$, when $n \geq 7$
**(C)** The sum $\sum_{n=-\infty}^{\infty} y[n] = 64$
**(D)** None of above is true

1.11 Impulse response of the filter is $h[n] = \sum_{k=-\infty}^{\infty} \big(2\delta[n-2k] - 2\delta[n-2k-1]\big)$
**(A)** Pole-zero-plot is in Figure 5
**(B)** Transfer function is $H(z) = \frac{2}{1-z^{-1}}$
**(C)** Filter is not stable
**(D)** Filter is of lowpass type

1.12 Consider a LTI filter $H(z) = 1 + 0.1z^{-10}$
**(A)** Pole-zero-plot of the filter is in Figure 6(a)
**(B)** Magnitude response $|H(e^{j\omega})|$ of the filter is in Figure 6(b)
**(C)** The length of impulse response $h[n]$ is 10
**(D)** Group delay $\tau(\omega)$ of the filter is constant

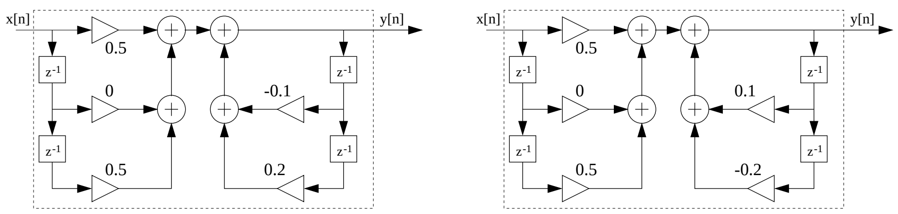


Figure 1: Problem 1.4, vaihtoehdot **(A)** ja **(B)** .

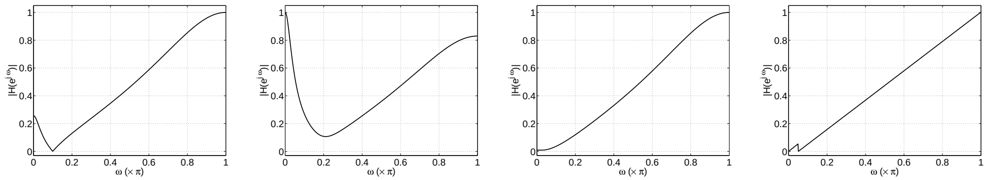


Figure 2: Problem 1.6, options **(A)** , **(B)** , **(C)** , **(D)** .

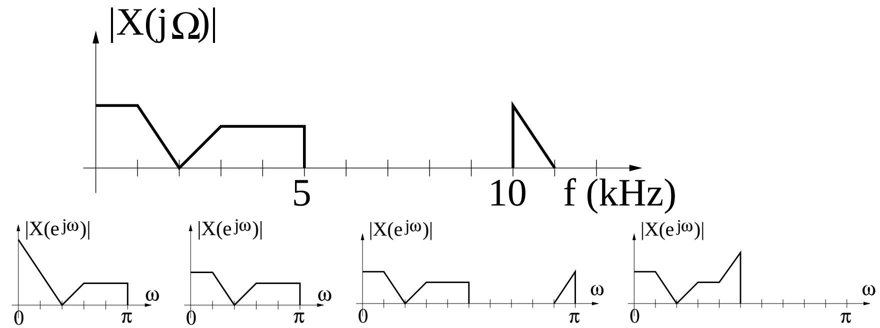


Figure 3: Problemn 1.8 kuvia. Top row: continuous $X(j\Omega)$, bottom row: options **(A)** , **(B)** , **(C)** , **(D)**

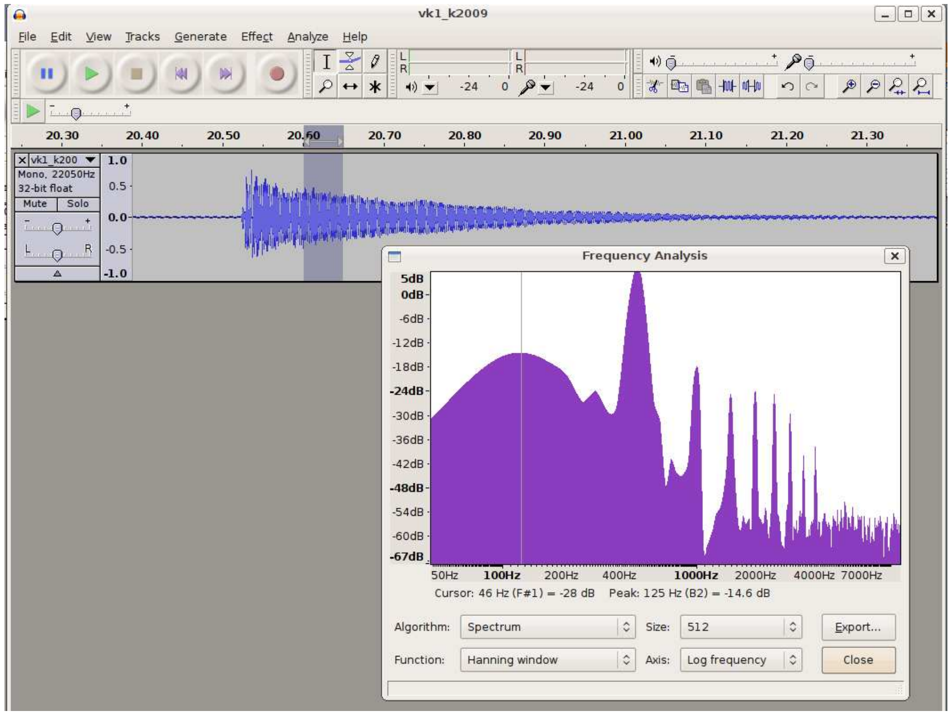


Figure 4: Problem 1.9, screenshot from Audacity.



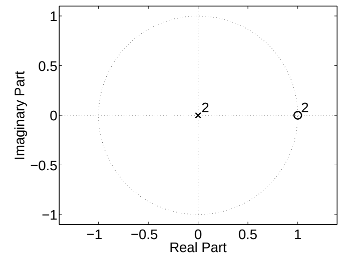


Figure 5: Problem 1.11, option **(A)** .

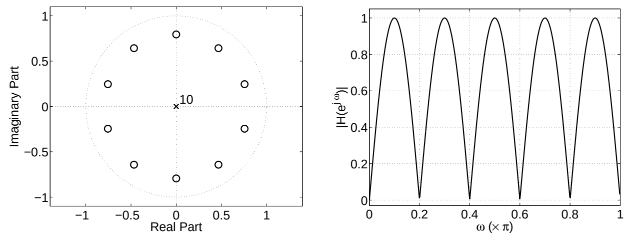


Figure 6: Problem 1.12, options **(B)** and **(D)** .

2) (6 p) In the first part of this course we have examined inputs $x[n]$, digital LTI systems $h[n]$ which process them and outputs $y[n]$. These can be processed both in time- and frequency-domain.

   Write down an essay about "filtering signals with digital LTI filters".

3) (1 p) Course feedback. Questionnaire `http://www.cis.hut.fi/Opinnot/T-61.3010/VK1_K2009/kyselyVK1_en.shtml` is open till 23-Mar 2009.