T-61.3010 Digital Signal Processing and Filtering

Mid term exam 2 / Final exam. Wed 7.5.2008 8-11. Hall M.

You can do MTE2 only once either 7.5. or 14.5. MTE2: Problems 1 and 2.

You can do final exam only once either 7.5. or 14.5. Final exam: Problems 2, 3, 4, 5, and 6. Begin each problem from a new page.

You can have a function calculator but not any math formula book. You will be given a course formula paper and a multichoice sheet for Problem 1 (MTE2).

1) (10 x 1p, max 8 p, **ONLY MTE2**) Multichoice There are 1-4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

   Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 8 and the minimum 0.

   Statements 1.1 – 1.4 are core of this course and straigt forward. Statements 1.5 – 1.10 need probably some computation on paper.

1.1 Causal and stable LTI filter

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 1.3z^{-1} + 0.4z^{-2}}$$

is depicted in a canonic (with respect to delays) direct form II in

(A) Figure 1(a)
(B) Figure 1(b)
(C) Figure 1(c)
(D) Figure 1(d)

1.2 Bilinear transform

(A) is bijection (one-to-one mapping), where half plane of analog s-plane is mapped to inside unit circle in z-plane
(B) one way to compute analog FIR-filter from corresponding digital filter
(C) is transform with which it is possible to shift quantization noise into a stopband of the filter
(D) is filtering stereo signal with a linear-phase filter

1.3 Window function used in digital FIR filter design

$$w[n] = 0.54 + 0.46 \cdot (\cos(\frac{2\pi n}{2M}))$$

(A) is ready FIR filter with normalize cut-off angular frequency $\omega_c = 2\pi \cdot M/N$, where N is filter order
(B) determines minimum attenuation in stopband as $20\log_{10}(M)$ decibels
(C) is typical Butterworth window, whose mainlobe length is $\Delta_{\mathrm{ML}} = 3.11\pi/M$
(D) can be used to cut the infinite-length impulse response $h_{\mathrm{ideal}}[n]$ into finite-length

1.4 When audio signal $x[n]$, whose sampling frequency is $f_T = 22050$ Hz and length about 0.40 seconds, is upsampled with factor $L = 2$

(A) number of samples will be about 44100
(B) sampling period will be $T = 0.40/22050$ Hz
(C) length of the signal will be still about 0.40 seconds with the new sampling frequency $f_T' = 44100$ Hz
(D) length of the signal will be about 0.80 seconds with the new sampling frequency $f_T' = 44100$ Hz

1.5 Examine digital IIR bandpass filter

$$H(z) = K \cdot \frac{2 - 0.108z^{-2} + 2z^{-4}}{1 + 1.16z^{-2} + 0.434z^{-4}}$$

whose maximum is reached at $\omega = \pi/2$. Determine the scaling factor K so that the maximum of the filter will be 1.

(A) $K \approx 0.067$
(B) $K \approx 0.50$
(C) $K \approx 0.67$
(D) $K \approx 15$

1.6 Stable analog filter $H(s) = \Omega/(s+\Omega)$, with prewarped $\Omega = k \cdot 0.5$, is changed to digital $H(z)$ using $s = k \cdot (1-z^{-1})/(1+z^{-1})$. The digital filter will be

(A) $H(z) = 1/(1+2k^{-1}z^{-1})$

(B) $H(z) = (1/3) \cdot (1+z^{-1})/(1-(1/3)z^{-1})$

(C) $H(z) = (1+z^{-1})/(1-3z^{-1})$

(D) $H(z) = (1-z^{-1})/(1.5-0.5z^{-1})$

1.7 Examine the second-order IIR filter with quantization block Q and first-order error-shaping in Figure 2(a). Let us write $w[n]$ and replace Q with noise source $e[n]$ as shown in Figure 2(b). Now you can write two difference equations, one $y[n] = \ldots$ and the other $w[n] = \ldots$.

Next, you can write down the output in frequency domain

$$Y(z) = H_x(z) \cdot X(z) + H_e(z) \cdot E(z)$$

where $H_x(z)$ is the actual filter and $H_e(z)$ is the filter for quantization error. These are

(A) $H_x(z) = \frac{1+1.8z^{-1}+0.82z^{-2}}{1+1.8z^{-1}+0.82z^{-2}}$ and $H_e(z) = \frac{kz^{-1}}{1+1.8z^{-1}+0.82z^{-2}}$

(B) $H_x(z) = \frac{1+1.8z^{-1}+0.82z^{-2}}{1-1.8z^{-1}+0.82z^{-2}}$ and $H_e(z) = \frac{1+kz^{-1}}{1-1.8z^{-1}+0.82z^{-2}}$

(C) $H_x(z) = \frac{1+1.8z^{-1}+0.82z^{-2}}{1-1.8z^{-1}+0.82z^{-2}}$ and $H_e(z) = \frac{1+(k-1.8)z^{-1}+0.82z^{-2}}{1-1.8z^{-1}+0.82z^{-2}}$

(D) None of pairs above is not true.

1.8 Continue from 1.7. Suppose quantization error as white noise, whose spectrum $E(z) = 1$. What is the best value for k, so that total noise $E_{\mathrm{TOT}}(z)$ is shifted from interesting passband.

(B) $k = -1$

(A) $k = 0$

(C) $k = 1$

(D) $k = 1.8$

1.9 Using command `[B, A] = cheby2(5, 30, 0.25);` you will get Chebychev II -type digital filter, whose order is $N = 5$, stopband minimum attenuation 30 dB and stopband cut-off $\omega_{\mathrm{stop}} = 0.25\pi$. The magnitude response of the filter is in

(A) Figure 3(a)

(B) Figure 3(b)

(C) Figure 3(c)

(D) Figure 3(d)

1.10 What can you do with a working piece of Matlab code below?

```
[x, fT] = wavread('mysignal.wav');
wL  = 256;
M   = length(x);
V   = zeros(ceil(M/wL), 1);
m   = 0;
for k = [1 : wL : M-wL]
   m    = m + 1;
   V(m) = sum(x(k : k+wL-1).^2);
end;
```

(A) Lowpass filter the signal with cut-off `wL` Hz

(B) Thresholding values of vector `V` it is possible to find out where the signal has silent parts

(C) Compute power spectrum values

(D) You can draw spectrogram from values of `V`

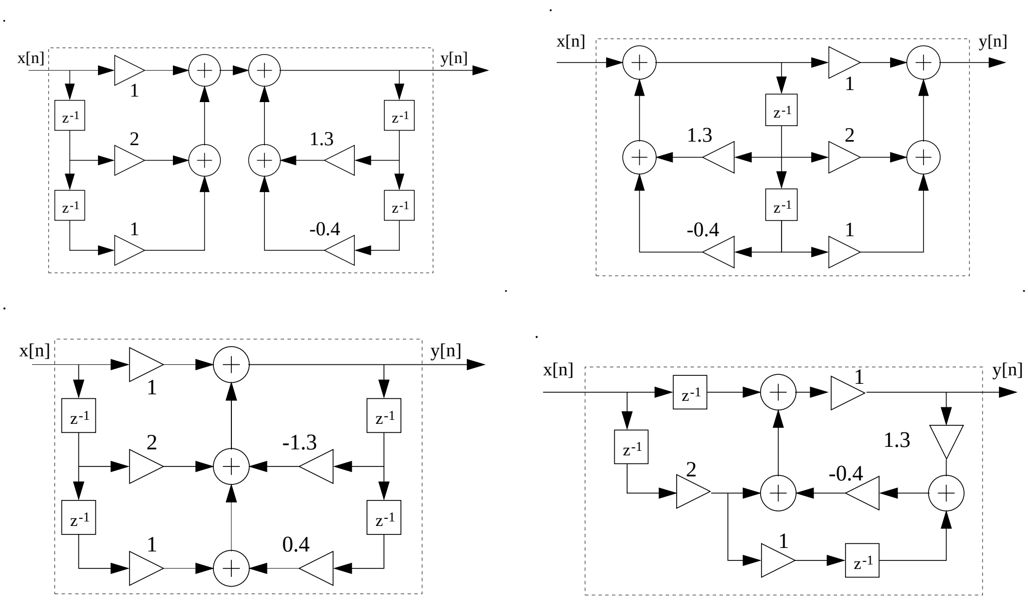


Figure 1: Multichoice statement 1.1 structures, top row **(A)** and **(B)** , bottom row **(C)** and **(D)** .

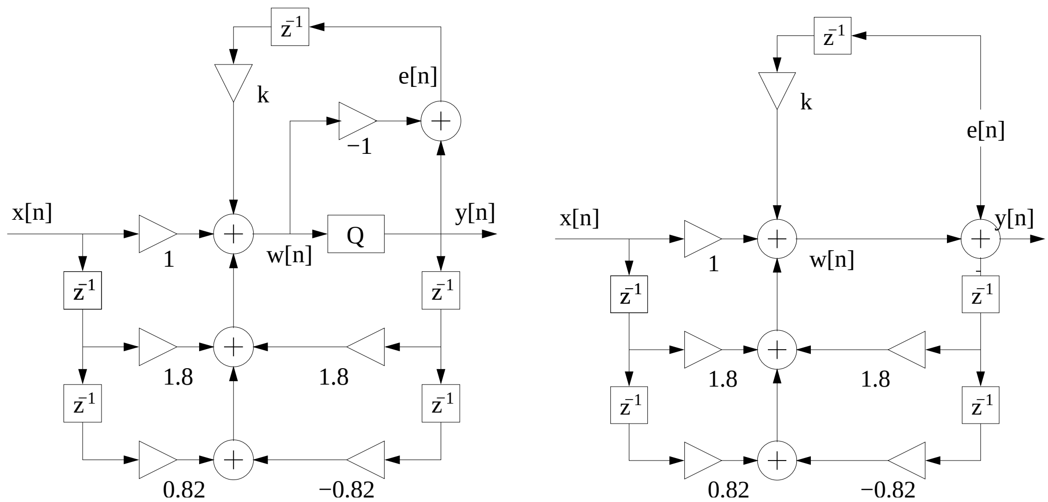


Figure 2: Multichoice statement 1.7 and 1.8, 2nd order IIR with 1st order error-feedback. Right, Q is replaced by noise source $e[n]$.

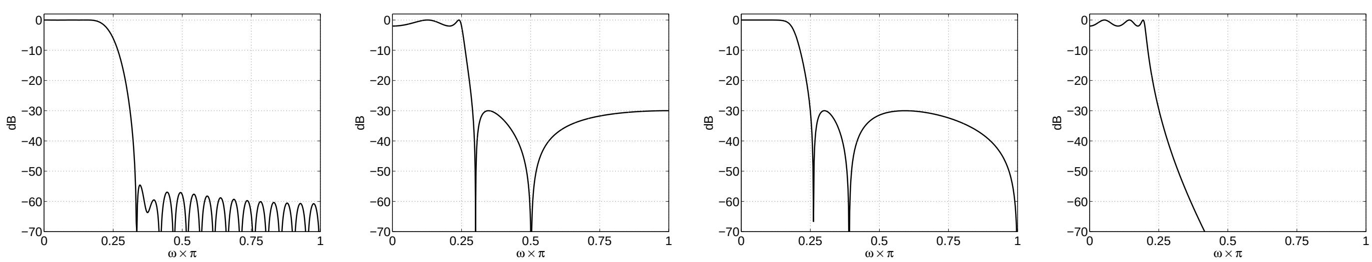


Figure 3: Multichoice statement 1.9 magnitude responses **(A)** , **(B)** , **(C)** , and **(D)** .

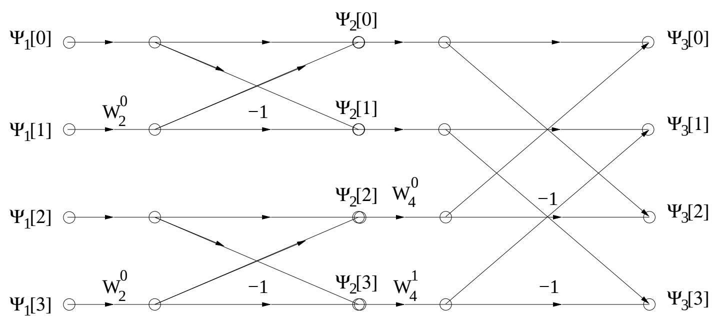


Figure 4: Problem 2A. Flow-diagram of “radix-2 DIT FFT”.

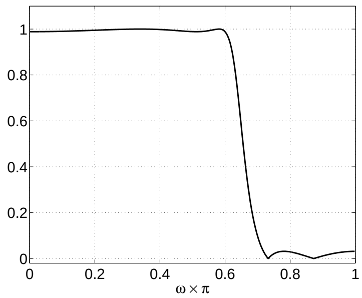
2) (6p, **MTE2 AND FINAL EXAM**) Choose either 2A or 2B.

2A) FFT algorithms. In addition to common properties you can also use an example of "radix-2 DIT FFT" algorithm, whose flow diagram is given in Figure 4 for 4 points and $r = 1, 2$, and $l_r = 0, \ldots, 2^{r-1} - 1$. Check out the butterfly equations and W_N from formula paper. Compute transform with intermediate steps for sequence $x[n] = 2\delta[n] + 4\delta[n-1] - \delta[n-2] + 5\delta[n-3]$.

2B) Examine stable and causal lowpass filter $H(z)$, whose passband ends at 3 kHz and whose frequency response is 10 kHz. Amplitude response is given in Figure 5(a) and first values of impulse response $h[n]$ in Figure 5(b).

a) Upsample filter with factor $L = 3$. Sketch both the new amplitude response $H^{'}(z) = H(z^3)$ and impulse response $h^{'}[n] = h[n/3]$ for first 10 values.

b) Finally, you'd like to have a lowpass filter with the same cut-off but with 30 kHz sampling frequency. What do you have to do? Sketch both the new amplitude response $|H^{''}(e^{j\omega})|$ and impulse response $h^{''}[n] = h[n/3]$ for first 10 values afterwards.



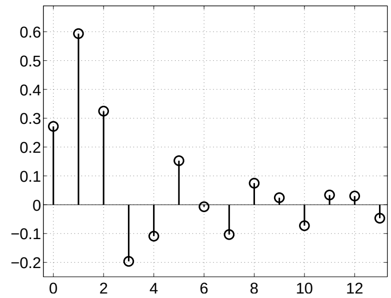


Figure 5: Problem 2B. Left, amplitude response. Right, impulse response.

3) (6p, **ONLY FINAL EXAM**) Examine cascade (series) of two LTI-systems $h[n] = h_1[n] \circledast h_2[n]$, where $h_1[n] = \delta[n] - 0.9\delta[n-1]$ and $h_2[n] = \delta[n] + 0.81\delta[n-2]$.

a) What is the impulse response $h[n]$ of the total system?

b) Write down the transfer function $H(z) = H_1(z) \cdot H_2(z)$.

c) Sketch the pole-zero-plot. (Hint: you can combine pole-zero-plots of $H_1(z)$ and $H_2(z)$ because they are terms of the product $H(z)$.)

d) Sketch the amplitude response of the filter. Give reasons why the maximum amplification is at $\omega = \pi$.

4) (6p, **ONLY FINAL EXAM**) Examine spectrum $X(j\Omega)$ of analog signal $x(t)$ in Figure 6.

a) What is the most important thing in Shannon's sampling theorem?

b) If the signal is sampled with $f_T = 10$ kHz, sketch the spectrum $X(e^{j\omega})$ of sampled sequence.

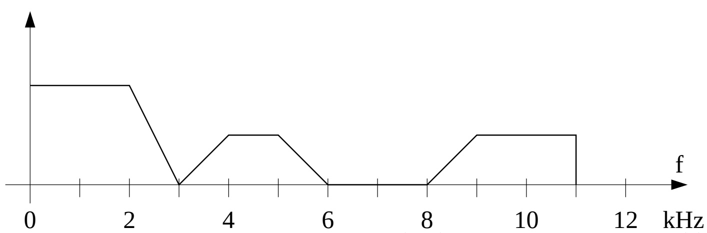


Figure 6: Problem 4. Spectrum $X(j\Omega)$ of analog signaln $x(t)$.

5) (6p, **ONLY FINAL EXAM**) Consider a stable and causeal LTI-filter

$$y[n] = x[n] - 4ax[n-1] + 9a^2x[n-2] + 1.2y[n-1] - 0.72y[n-2]$$

whose coefficient a is real value determined later.

a) Draw the block diagram of the filter in a canonic (with respect to delays) direct form II structure.

b) Determine $H(z)$.

c) Examine frequency properties of the filter as a function of a using pole-zero-plot, when $a \geq 0$.

6) (6p, **ONLY FINAL EXAM**) Digital IIR filter design.