

T-61.3010 Digital Signal Processing and Filtering

2nd mid term exam / final exam, Tue 15-May-2007 at 16-19. Hall A.

If you are doing the **2nd mid term exam**, reply to problems 1-2. You are allowed to do 2nd MTE **only once either 7.5. or 15.5.**

If you are doing the **final exam**, reply to problems 2-5. You are allowed to do final exam **only once either 7.5. or 15.5.**

Write down on first paper, if you are doing **mid term exam or final exam!**

You are not allowed to use any math table books of your own. A table of formulas is delivered as well as a form for multichoice problem (either 1 or 5). A (graphical) calculator is allowed.

Start a new task from a **new page**. Write all **intermediate steps**.

- 1) (**ONLY 2nd MTE**, 14 x 1p, max 12 p) Multichoice. There are 1-4 correct answers in the statements, but choose **one and only one**. Fill in your solutions in a specific **form**.

Correct answer +1 p, wrong answer -0.5 p, no answer 0 p. You do not need write why you chose your option. Reply to as many as you want. The maximum number of points is 12, and the minimum 0.

- 1.1 In which cases the filter has linear phase response?

(A) Finite-length impulse response is symmetric, i.e., $h[n] = h[N - n]$

(B) Zeros of the real-coefficient FIR filter are on unit circle

(C) In case of FIR filter, for each pair of zeros inside the unit circle $z_{1,2} = r \cdot e^{\pm j\theta}$ there exists a corresponding pair of zeros outside the unit circle $z_{3,4} = (1/r) \cdot e^{\pm j\theta}$, when there are no zeros at the unit circle, nor in real axis

(D) The length of the impulse response of the filter is finite

- 1.2 The LTI filter in Figure ??(a)

(A) is of structure type Direct Form II

(B) is FIR filter

(C) has the transfer function

$$H(z) = \frac{2 + 0.25z^{-1}}{1 + 0.25z^{-1} - 0.375z^{-2}}$$

(D) the value of the impulse response $h[n]$ at $n = 0$ is $h[0] = 1$

- 1.3 Consider a LTI filter with transfer function

$$H(z) = \frac{1 + 0.5z^{-1}}{1 - 0.2z^{-1} + 1.2z^{-2}}$$

where region of convergence (ROC) is "normal", that is, the outer part of the out-most pole.

(A) Filter $H(z)$ has been drawn in Figure 1(b) canonic with respect to delays in Direct Form II

(B) It is an allpass filter

(C) If it is expressed in Direct Form I, then it has two delay registers (z^{-1})

(D) Filter is stable

- 1.4 LTI-filter in Figure 2(a) with constants A and B ,

(A) can be linear-phase if choosing A and B correctly from a set of positive integers

(B) the transfer function is

$$H(z) = [1 + Az^{-1}]/[1 - (A + B)z^{-1}]$$

(C) the transfer function is

$$H(z) = [1 + Az^{-1}]/[B(1 + A)z^{-1}]$$

(D) none of choices above is correct

- 1.5 Consider a block diagram in Figure 2(b), where there are three LTI systems $S(z)$, $R(z)$, and $T(z)$. The transfer function of the system is

$$(A) H(z) = \frac{S(z) + R(z)T(z)}{1 - T(z)}$$

$$(B) H(z) = \frac{S(z)R(z)}{1 - S(z)T(z)}$$

$$(C) H(z) = \frac{R(z)S(z)}{1 + S(z)T(z)}$$

$$(D) H(z) = \frac{R(z)S(z)}{T(z)}$$

- 1.6 Analog filter $H(s) = \Omega/(s + \Omega)$, where prewarped cut-off frequency $\Omega = k \cdot 0.5$, is converted to digital $H(z)$ using bilinear transform. The digital filter is

$$(A) H(z) = (1/3) \cdot [1 + z^{-1}]/[1 + 0.5z^{-1}]$$

$$(B) H(z) = (1/3) \cdot [1 + z^{-1}]/[1 - (1/3)z^{-1}]$$

$$(C) H(z) = 0.5/[0.5 + z^{-1}]$$

$$(D) H(z) = [1 + z^{-1}]/[1.5 - z^{-1}]$$

- 1.7 The oscillation in the passband of a FIR filter is defined in linear axis so that amplitude response may vary between 0.9 and 1.1, see Figure 3(a), left y-axis.

If the maximum value of the amplitude response is first scaled to 1, which corresponds 0 dB in logarithmic decibel scale, then what is the maximum power (square) oscillation in passband? In other words, what is the value for α_{max} in Figure 3(a), right y-axis? Hint: Formulae "Logarithm".

(A) $\alpha_{max} \approx 1.74$ dB

(B) $\alpha_{max} \approx 3.01$ dB

(C) $\alpha_{max} \approx 20.0$ dB

(D) $\alpha_{max} \approx 40.0$ dB

- 1.8 Design a digital elliptic lowpass filter in Matlab. Given the specifications, the minimum order of the filter is $N = 11$ (`e11ipord`). If the coefficient vectors \mathbf{B} and \mathbf{A} of the filter are computed (`e11ip`) using order $N = 22$,

(A) filter does not meet given specifications any more

(B) filter becomes too heavy to implement and too heavy to use in practice

(C) transition band becomes narrower

(D) cut-off frequency (frequencies \mathbf{w}_p , \mathbf{w}_s) grow twice as big

- 1.9 You want to compute discrete Fourier transform (DFT-128) of a 128-length sequence $x[n]$ (variable x), and draw a spectrum $|X(e^{j\omega})|$. Which sequence of commands has produced a spectrum in Figure 3(b)?

(A) `xF = fft(x); w = [0 : pi]; plot(w, abs(xF));`

(B) `xF = fft(x); f = [0 : pi/128 : 2]; stem(f, 20*log10(abs(xF)));`

(C) `xF = fft(x); M = length(xF); f = 2 * [0 : M-1] / M; plot(f, 20*log10(abs(xF)));`

(D) `xF = fft(x); w = [0 : pi/64 : 2*pi-pi/128]; plot(abs(xF));`

- 1.10 The ideal highpass filter $H_{HP}(z)$ has cut-off frequency at $\omega_c = \pi/4$. Now

(A) $h_{HP}[0] = 0$

(B) $h_{HP}[0] = 0.25$

(C) $h_{HP}[0] = 0.75$

(D) $h_{HP}[0] = 1$

- 1.11 Limit cycle of the filter (related to finite wordlength)

(A) is a fundamental part of the multirate systems

(B) is a mapping from a stable analog filter $H(s)$ to a stable digital filter $H(z)$

(C) does not exist for FIR filters

(D) makes IIR filters impractical, therefore, IIR filters are never used in real life, but FIR filters

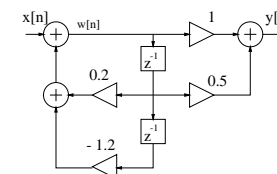
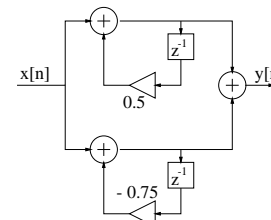


Figure 1: (a) and (b): Figures for multichoice statements 1.2 and 1.3.

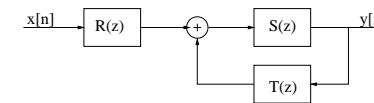
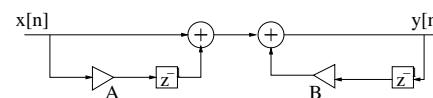


Figure 2: (a) and (b): Figures for multichoice statements 1.4 and 1.5.

- 1.12 A periodic sequence ($N_0 = 3$) $x[n] = \{\dots, 1, -2, \dots\}$ is fed into a digital system

$$x[n] \rightarrow \boxed{\uparrow 3} \rightarrow \boxed{\downarrow 4} \rightarrow y[n]$$

What comes out?

(A) $y[n] = \{\dots, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, \dots\}$, where period $N_0 = 12$

(B) $y[n] = \{\dots, 1, 0, 0, 1, 0, 0, -2, 0, 0, \dots\}$, where period $N_0 = 9$

(C) $y[n] = \{\dots, 1, -2, 0, \dots\}$, where period $N_0 = 4$

(D) none of choices above is correct

- 1.13 Consider a sequence $x[n]$ in top row of Figure 4. The sequence $x[n]$ is fed into multirate system

$$x[n] \rightarrow \boxed{\downarrow 4} \rightarrow y[n].$$

Which figure does correspond to the output spectrum $|Y(e^{j\omega})|$?

(A) First on bottom row of Figure 4

(B) Second on bottom row of Figure 4

(C) Third on bottom row of Figure 4

(D) Fourth on bottom row of Figure 4

- 1.14 The sampling frequency of a digital sequence is to be increased to $(3/2)$ of the original. When having proper "anti-alias" and "anti-imaging" filters $H_i(z)$, what is the correct way to do this?

$$(A) x[n] \rightarrow \boxed{H_D(z)} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\uparrow 3} \rightarrow y[n]$$

$$(B) x[n] \rightarrow \boxed{\uparrow 3} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{H_I(z)} \rightarrow y[n]$$

$$(C) x[n] \rightarrow \boxed{\uparrow 3} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 2} \rightarrow y[n]$$

$$(D) x[n] \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{H(z)} \rightarrow \boxed{\uparrow 3} \rightarrow y[n]$$

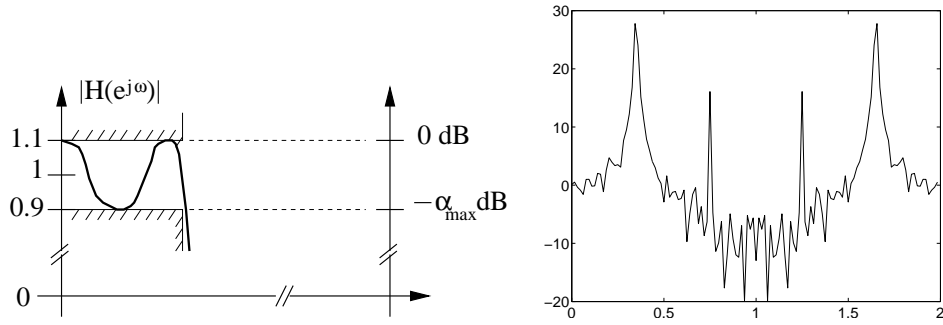


Figure 3: (a) and (b): Figures for multichoice statements 1.7/5.13 and 1.9.

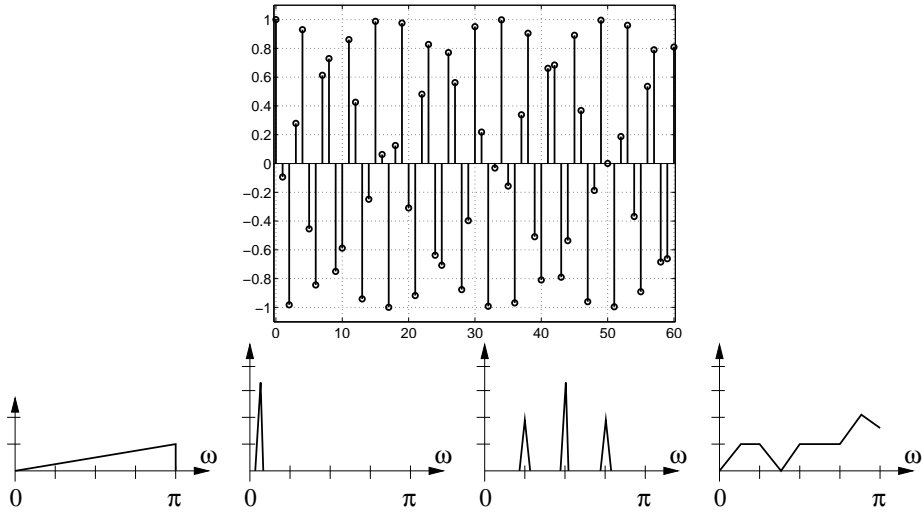


Figure 4: Top row: sequence $x[n]$, bottom row: (A) , (B) , (C) , (D) : Figures for multichoice statement 1.13.

2) (MTE and FINAL EXAM, 6p) Essay: Window functions and the usage of them.

Instructions: Write down clearly with big enough font. Divide your text into paragraphs. If you draw figures, remember to explain them.

3) (ONLY FINAL EXAM, 6p) A LTI filter can be expressed with its zeros and poles as

$$H(z) = G \cdot \frac{\prod_{m=1}^M (1 - d_m z^{-1})}{\prod_{n=1}^N (1 - p_n z^{-1})}$$

given in the formula paper, section "LTI filter analysis". Consider a causal and stable IIR filter whose zeros are at $d_1 = 0.9 + 0.1j$, $d_2 = 0.9 - 0.1j$, $d_3 = -0.9 + 0.1j$, and $d_4 = -0.9 - 0.1j$, and poles are at $p_1 = -0.1 + 0.9j$, $p_2 = 0.1 + 0.9j$, $p_3 = -0.1 - 0.9j$, and $p_4 = 0.1 - 0.9j$. The filter structure is drawn in Figure 5(a).

- Draw the pole-zero plot and sketch the magnitude response $|H(e^{j\omega})|$ in range $0 \dots \pi$.
- Determine the transfer function $H(z)$ in the format

$$H(z) = K \cdot \frac{1 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_N z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_M z^{-N}}$$

where the coefficients $b_i, a_i \in \mathbb{R}$. The gain K is defined so that the maximum of the magnitude response is 1. Suppose here that the passband is flat (even if this is not exactly true) and the maximum of $|H(e^{j\omega})|$ is reached at the middle point of the passband. What are the values N and M ? What is the filter order?

Consider now the effects of the finite wordlength. The same filter structure with second-order error-shaping (error feedback) filter with coefficients k_1 and k_2 is given in Figure 5(b).

The input values are represented with B bits with fixed-point arithmetics. After multiplications the number of bits is $2B$. In order to get the number of bits in output to B , it is necessary to quantize values of $w[n]$ (block Q). Quantization error can be compensated using error feedback (error-shaping filter).

- Write down a set of difference equations that describe the computation of the whole system with error feedback.
- Assume that the quantization noise $\epsilon[n]$ is white noise, i.e., the spectrum of that is flat $E(z) = 1$ for all frequencies. How does the spectrum of the total noise $E_{tot}(z) = H_e(z)E(z)$ look like (sketch!) if there is no compensation, i.e., $k_1 = 0$ and $k_2 = 0$.
- Finally, derive simple values for k_1 and k_2 so that the effect of the total noise is suppressed in the passband.

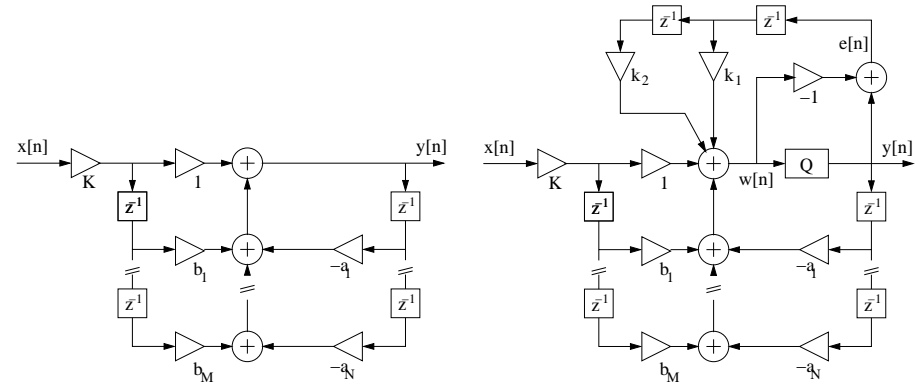


Figure 5: Problem : (a) Filter $H(z)$. Notice that only a part of coefficients has been drawn. (b) Filter with quantization and second-order error feedback.

4) (ONLY FINAL EXAM, 6p) Examine the cascade system of three LTI systems in Figure 6. It is known that

$$h_2[n] = \delta[n - 2] - \delta[n - 3]$$

and the whole cascade system

$$h[n] = \delta[n] - 7\delta[n - 1] + 19\delta[n - 2] - 25\delta[n - 3] + 16\delta[n - 4] - 4\delta[n - 5]$$

- What is the impulse response of $h_1[n]$? Remember to give the intermediate steps.
- Draw the pole-zero-plots of $h_1[n]$ and $h_2[n]$ and sketch also the corresponding amplitude responses $|H_i(e^{j\omega})|$.
- Show that the filter $h_2[n]$ is linear-phase. Compute also the group delay $\tau(\omega)$.

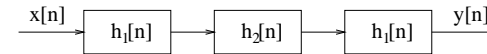


Figure 6: Problem 4 cascade system

- 5) (ONLY FINAL EXAM, 14 x 1p, max 12 p) Multichoice. There are 1-4 correct answers in the statements, but choose **one and only one**. Fill in your solutions in a specific **form**.

Correct answer +1 p, wrong answer -0.5 p, no answer 0 p. You do not need write why you chose your option. Reply to as many as you want. The maximum number of points is 12, and the minimum 0.

- 5.1 Examine discrete-time system $y[n] = 3x[3n]$:
- (A) system is linear
 (B) system is time-invariant
 (C) system is causal
 (D) none of choices above is correct
- 5.2 The fundamental normalized angular frequency ω_0 of the sequence $x[n] = \cos(0.2\pi n) + 2\sin(0.4\pi n) - \sin(0.5\pi n)$ is
- (A) $\omega_0 = 0.1\pi$
 (B) $\omega_0 = 0.2\pi$
 (C) $\omega_0 = 0.5\pi$
 (D) $\omega_0 = 2\pi$
- 5.3 The infinite-length impulse response of a causal LTI filter is $h[n] = 0.5\delta[n] - 0.5\delta[n-1] + 0.5\delta[n-2] - 0.5\delta[n-3] + 0.5\delta[n-4] - 0.5\delta[n-5] + \dots$
- (A) filter is stable
 (B) filter is FIR
 (C) there is a feedback loop in the filter structure
 (D) none of choices above is correct
- 5.4 Compute the linear convolution $y[n] = h[n] \otimes x[n]$? of sequences $h[n] = \delta[n] + \delta[n-2]$ and $x[n] = -\delta[n] + \delta[n-1] - 2\delta[n-2]$
- (A) $y[n]$ is 6 units long
 (B) $y[n] = -\delta[n] + \delta[n-1] - 2\delta[n-2] + \delta[n-3] + 2\delta[n-4]$
 (C) $y[n] = -\delta[n] + \delta[n-1] - 3\delta[n-2] + \delta[n-3] - 2\delta[n-4]$
 (D) none of choices above is correct
- 5.5 Consider a causal and stable transfer function

$$H(z) = \frac{1+z^{-3}}{1+0.2z^{-1}}$$

What is the value of the inverse transform $h[n]$ at $n = 4$ with three significant digits?

- (A) $h[4] \approx -0.198$
 (B) $h[4] \approx 0.00960$
 (C) $h[4] \approx 0.202$
 (D) $h[4] \approx 1.67$
- 5.6 See the spectrum $|X(j\Omega)|$ of a continuous-time real signal in the top row of Figure 7. When sampling with $f_T = 10$ kHz, the output spectrum of the sequence $x[n]$ is in the bottom row of Figure 7
- (A)
 (B)
 (C)
 (D)
- 5.7 Consider a filter

$$H(z) = \frac{1+0.2z^{-1}}{1+0.2z^{-1}-0.35z^{-2}}$$

- (A) poles are complex conjugates $p_1 = re^{j\theta}$ and $p_2 = re^{-j\theta}$
 (B) filter order is 3
 (C) the other pole is at $p_1 = -0.7$
 (D) a zero is at $z_1 = 0.2$

- 5.8 Consider a LTI filter, whose difference equation is $y[n] - 0.3y[n-1] + 0.8y[n-2] = x[n] + 0.1x[n-4]$.
- (A) there are two and only two zeros inside the unit circle
 (B) there are two and only two poles inside the unit circle
 (C) there are four and only four poles outside the unit circle
 (D) the filter order is two

- 5.9 The frequency response of a filter is $H(e^{j\omega}) = 0.5e^{4j\omega} + 1$
- (A) Impulse response is 4 samples long
 (B) Zeros of the filter are on the same circle with equal intervals
 (C) The maximum of the magnitude is at $\omega = \pi/4$
 (D) Filter is causal

- 5.10 Minimum-phase filter
- (A) is always FIR
 (B) poles are on unit circle
 (C) all zeros are in origin
 (D) none of choices above is correct
- 5.11 Filter

$$H(z) = 1 + 0.2z^{-1} - 0.2z^{-2} + 0.04z^{-3} - 0.2z^{-4}$$

can be implemented as a polyphase structure

$$H(z) = H_1(z^3) + z^{-1}H_2(z^3) + z^{-2}H_3(z^3)$$

where

- (A) $H_1(z) = 1.04$, $H_2(z) = 0$, and $H_3(z) = -0.2$
 (B) $H_1(z) = 1 + 0.04z^{-1}$, $H_2(z) = 0.2 - 0.2z^{-1}$, and $H_3(z) = -0.2$
 (C) $H_1(z) = 1 - 0.2z^{-2}$, $H_2(z) = 0.2 + 0.04z^{-2}$, and $H_3(z) = -0.2z^{-2}$
 (D) $H_1(z) = 1 + 0.04z^{-3}$, $H_2(z) = 0.2z^{-1} - 0.2z^{-4}$, and $H_3(z) = -0.2z^{-2}$
- 5.12 Analog filter $H(s) = \Omega/(s+\Omega)$, where prewarped cut-off frequency $\Omega = k \cdot 0.5$, is converted to digital $H(z)$ using bilinear transform. The digital filter is
- (A) $H(z) = (1/3) \cdot [1 + z^{-1}]/[1 + 0.5z^{-1}]$
 (B) $H(z) = (1/3) \cdot [1 + z^{-1}]/[1 - (1/3)z^{-1}]$
 (C) $H(z) = 0.5/[0.5 + z^{-1}]$
 (D) $H(z) = [1 + z^{-1}]/[1.5 - z^{-1}]$

- 5.13 The oscillation in the passband of a FIR filter is defined in linear axis so that amplitude response may vary between 0.9 and 1.1, see Figure 3(a), left y-axis.

If the maximum value of the amplitude response is first scaled to 1, which corresponds 0 dB in logarithmic decibel scale, then what is the maximum power (square) oscillation in passband? In other words, what is the value for α_{max} in Figure 3(a), right y-axis? Hint: Formulae "Logarithm".

- (A) $\alpha_{max} \approx 1.74$ dB
 (B) $\alpha_{max} \approx 3.01$ dB

- (C) $\alpha_{max} \approx 20.0$ dB
 (D) $\alpha_{max} \approx 40.0$ dB

- 5.14 The sampling frequency of a digital sequence is to be increased to $(3/2)$ of the original. When having proper "anti-alias" and "anti-imaging" filters $H_i(z)$, what is the correct way to do this?

- (A) $x[n] \rightarrow \boxed{H_D(z)} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\uparrow 3} \rightarrow y[n]$
 (B) $x[n] \rightarrow \boxed{\uparrow 3} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{H_I(z)} \rightarrow y[n]$
 (C) $x[n] \rightarrow \boxed{\uparrow 3} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 2} \rightarrow y[n]$
 (D) $x[n] \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{H(z)} \rightarrow \boxed{\uparrow 3} \rightarrow y[n]$

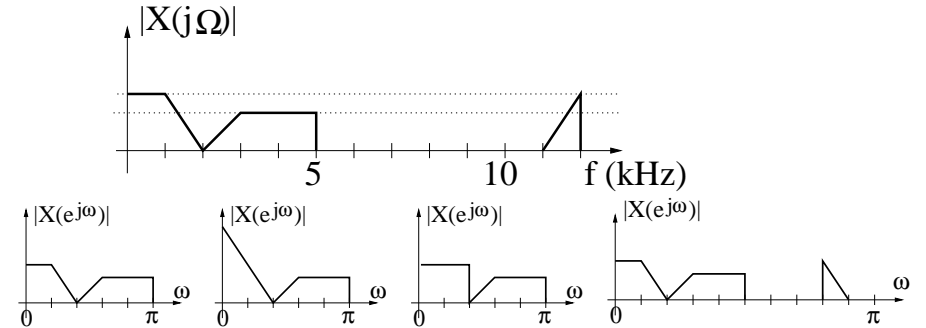


Figure 7: Figures for multichoice statement 5.6. Top row: continuous $X(j\Omega)$, Bottom row: choices (A) , (B) , (C) , (D)