Tik-61.145 Principles of digital signal processing

1st partial test March 12, 1996

1. (4p) Consider an analog signal consisting of five frequency components. The spectrum of the signal showing the amplitudes of the frequency components is in figure 1. The spectrum is symmetric around the origin; only positive frequencies are shown.

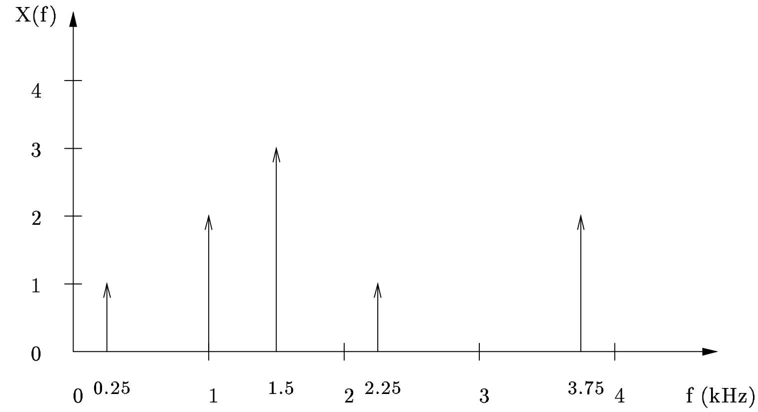

Figure 1: Spectrum of the analog signal.

   (a) The signal is sampled using 4 kHz sampling frequency. Determine and sketch the spectrum of the digital signal on the band 0...2 kHz.

   (b) Before sampling, the signal is filtered using analog antialiasing filter. The magnitude response of the filter is:

$$|H(f)| = \begin{cases} 1, & 0 \leq f \leq 1.75 \text{ kHz} \\ 0.1, & f \geq 2.25 \text{ kHz} \end{cases}$$

   The filter has finite transition band $1.75 < f < 2.25$ kHz.

   Determine and sketch the spectrum of the digital signal on the band 0...2 kHz, when the sampling frequency is the same 4 kHz.

2. (6p) Consider pole/zero diagrams (a)-(c) and magnitude responses (1)-(3) of figure 2. The sampling frequency is 10 kHz; the frequency band shown in the magnitude responses is 0...5 kHz ($\omega = 0 \ldots \pi$).

   (a) Combine pole/zero diagrams with corresponding magnitude responses. Explain your choices briefly. If there are pole/zero diagrams with no corresponding magnitude responses, explain the reason for this.

   (b) What are the orders of the transfer functions (filters) corresponding to pole/zero diagrams?

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(a) (1)

(b) (2)

(c) (3)

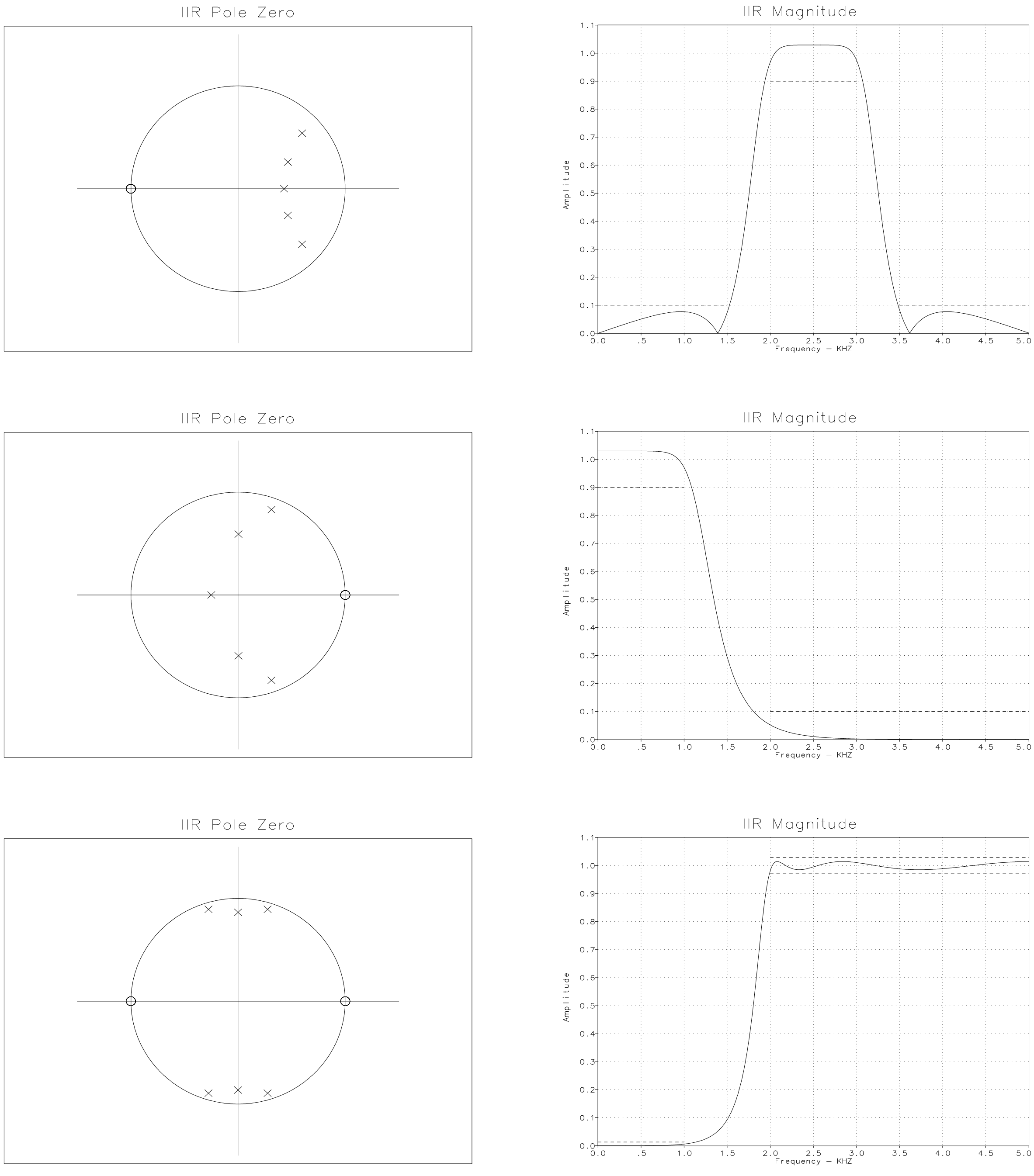

Figure 2: Pole/zero diagrams and magnitude responses.

3. (8p) Consider a discrete-time system with following input-output relationship ($x(n)$ is the input, and $y(n)$ is the output):

$$y(n) = x(n) - x(n-2) + \frac{1}{4}y(n-2).$$

(a) Determine $H(z)$, the (z-plane) transfer function of the system.

(b) Compute poles and zeros of $H(z)$. Sketch the pole/zero diagram in the z-plane.

(c) Determine $H(e^{j\omega T})$, the frequency response of the system. Sketch the magnitude response $|H(e^{j\omega T})|$. What kind of filter is this?

(d) Write $H(z)$ as a combination of two filter blocks in the following way:

$$H(z) = H_1(z)H_2(z),$$

where $H_1(z)$ and $H_2(z)$ are the transfer functions of the first order blocks.

(e) Sketch $|H_1(e^{j\omega T})|$ and $|H_2(e^{j\omega T})|$, the magnitude responses of the first order blocks. What kind of filters are the blocks, when they are considered separately?

4. (6p) Input into a linear, transfer invariant and causal discrete system was $x(n)$. The input produced following output:

n	$x(n)$	$y(n)$
0	1	3
1	2	5
2	-1	?
3	0	?
4	2	?

(a) Determine $h(n)$, the impulse response of the system, using $x(n)$ and $y(n)$. It is known that $h(n)$ is of form (a, b are unknown constants):

$$h(n) = \begin{cases} 0, & \text{when } n < 0 \\ a, & \text{when } n = 0 \\ b, & \text{when } n = 1 \\ 0, & \text{when } n > 1 \end{cases}$$

(b) Compute three next output values ($y(2)$, $y(3)$ and $y(4)$).