Tik-61.146 Digital Signal Processing and Filtering
Exam/ replacement of a *missing* partial test Dec 11, 1998

1st partial test: exercises 1,2,3,4

2nd partial test: exercises 5,6,7,8

Exam: exercises 2,4,5,6,7

Note! Indicate clearly, which test you are going to do. You may do only one of the three tests.

1. (6p) Are the following statements true or false? (Correct answer: +1p, no answer 0p, wrong answer: −1p.)

   (a) multiplication in the frequency domain corresponds to convolution in the time domain

   (b) a causal filter is stable only if its zeroes lie inside the unit circle

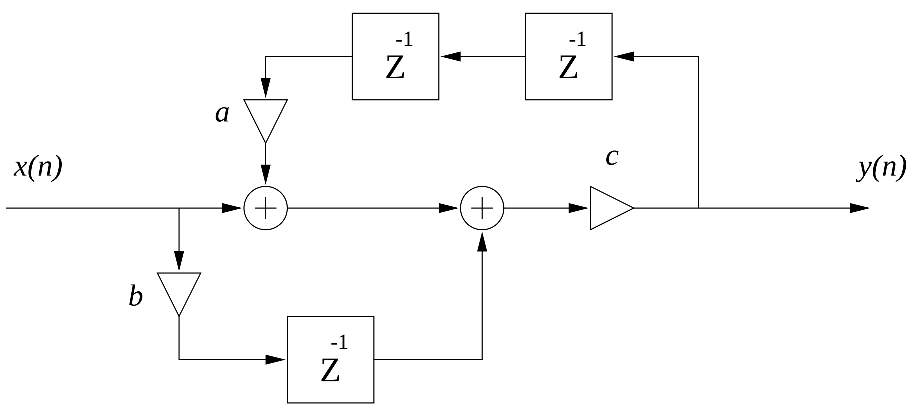
   (c) stable filters are always causal

   (d) if f_s is the sampling frequency, the frequencies on the interval $f_s/2 \ldots f_s$ are aliased on the interval $0 \ldots f_s/2$

   (e) discrete Fourier transform corresponds to z-transform on the imaginary axis of the z-plane

   (f) The implementation of FIR filter cannot have feedback loops (no recursion). Otherwise the impulse response would'nt be finite.

2. (6p) In Figure 1 there is a flow diagram of a causal filter. The filter parameter values are a=−1/16, b=−1 and c=1.



Kuva 1: Flow diagram of a filter.

   (a) Find the difference equation and transfer function of the system.

   (b) Compute poles and zeroes of the system and sketch the pole-zero -diagram. Is the system stable?

   (c) Compute and sketch the magnitude response of the filter. Which frequency has the minimum gain? What kind of a filter is this?

   (d) Can the stability of the system be affected by modifying the value of parameter c? Motivate your answer.

3. (6p)

Are the following systems linear, shift-invariant and/or causal? The systems are defined eihter using a difference equation or impulse response. Motivate your answer.

(a) $y(n) = nx(n)$

(b) $y(n) = x(n) + n$

(c) $h(n) = 1/3[\delta(n-1) + \delta(n) + \delta(n+1)]$

4. (6p) Consider the following two systems.

The impulse response $h_1(n)$ ($u(n)$ is the unit step function) of the system 1 is

$$h_1(n) = \left(\frac{1}{2}\right)^{n-1} u(n).$$

The transfer function $H_2(z)$ of the system 2 is

$$H_2(z) = \frac{4}{2 - z^{-1}}, \quad |z| < \frac{1}{2}.$$

Is system 1 or 2 equivalent to a causal system (motivate shortly), whose transfer function is

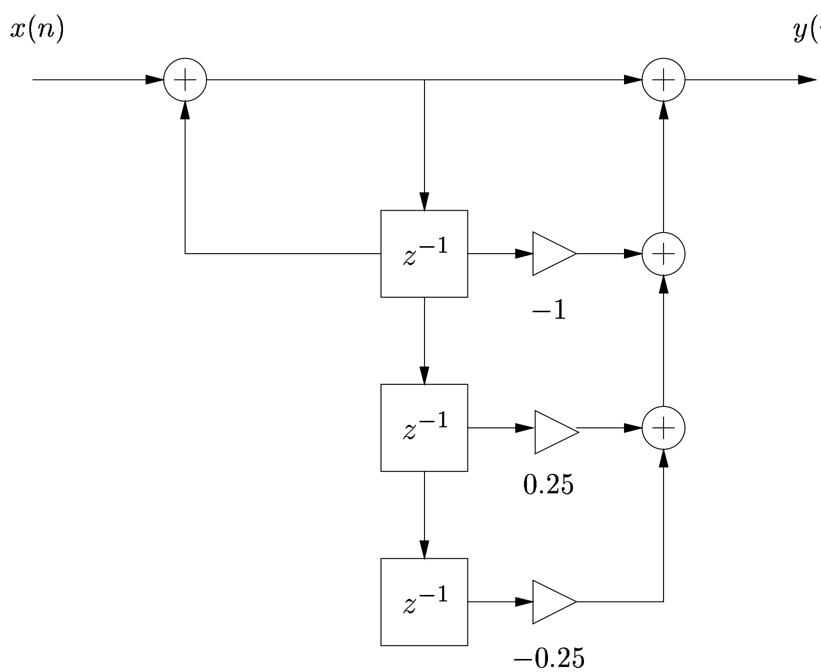
$$H(z) = \frac{2}{1 - \frac{1}{2}z^{-1}}?$$

5. (6p) Are the following statements true or false? (Correct answer: +1p, no answer 0p, wrong answer: −1p.)

   (a) FIR filters have always linear phase responses.
   (b) Elliptic IIR filter has sharper transition bands compared with filters of same order designed with other IIR filter design algorithms.
   (c) The scaling of signal in order to suppress the overflows in a filter also gives a better signal-to-noise ratio in the filter.
   (d) Bilinear transform causes folding of magnitude response in the digitalization of analog filters.
   (e) Combination of cascaded FIR and IIR filter is never stable.
   (f) Hanning-window (in spectrum analysis) has better suppression of the sidelobes and narrower mainlobe than the rectangular window.

6. (6p) Consider the filter realization in Figure 2.

   (a) Determine $H(z)$, the transfer function of the filter corresponding to the figure. What is the order of the transfer function?
   (b) Is the filter type IIR of FIR? Why?
   (c) Compute the poles and zeros and sketch the pole-zero diagram in the z-plane. Sketch the amplitude response.
   (d) Realize the filter in the simplest possible form. Draw the signal flow graph.



Kuva 2: Suodinrakenne.

7. (6p) Consider a filter whose input/output relationship is defined by difference equation

$$y(n) = x(n) + ay(n-1), \ a = 0.5.$$

(a) Compute the impulse response of the filter. Is the filter stable?

(b) Study the impulse response of the filter using four bit precision (sign bit and three bits) and rounding arithmetic. Multiplications are rounded so that the three least significant bits are dropped out, and if the first ignored bit was 1, the least significant bit is added 1. It is assumed that the numbers have been scaled so that their absolute values are less than one. For example, $\delta(0) = (0.111)_2 = 7/8$ (the greatest positive number) and $a = 0.5 = (0.100)_2 = 1/2$.

(c) Determine (by looking at the impulse response) if the filter is now stable or not. Motivate your answer.

(d) Is the filter in (b) linear? Motivate your answer.

8. (6p) We wish to reduce the sampling rate for a discrete-time signal $x(n)$ to two-thirds the original rate ω_s. The band of interest (narrowband signal) is $[0, \frac{\omega_s}{4}]$. Sketch the appropriate system. Determine the stopband edge frequency ω_r for aliasing suppression filter.