Tik-61.246 Digital Signal Processing and Filtering

Final / Mid Term Exam 18.12.2000 at 12-15. Halls B and C.
You may use a (graphical) calculator and a mathematical reference book. Storing additional material into the calculator's memory is strictly forbidden.

Write down which exam you are taking: final exam, 1st or 2nd mid term exam

Problems:	**1st mid term exam:**	**1, 2, 3 and 4**
	2nd mid term exam:	**5, 6 and 7**
	final exam:	**3, 4, 5, 6 and 7**

1. (3p) Are the following filters 1) linear, 2) shift-invariant, 3) causal, 4) stable? Explain briefly either with words or mathematical formulas.

   a) $y[n] = x[n+1] + x[n-1] - 2$

   b) $y[n] = 3x[3n]$

   c) $y[n] = x[n] + 2y[n-1]y[n-2]$

2. (3p) Figure 1 shows the zero-pole diagrams of two LTI systems. Sketch the corresponding amplitude responses. Are the filters of FIR or IIR type? What are the orders of the filters?

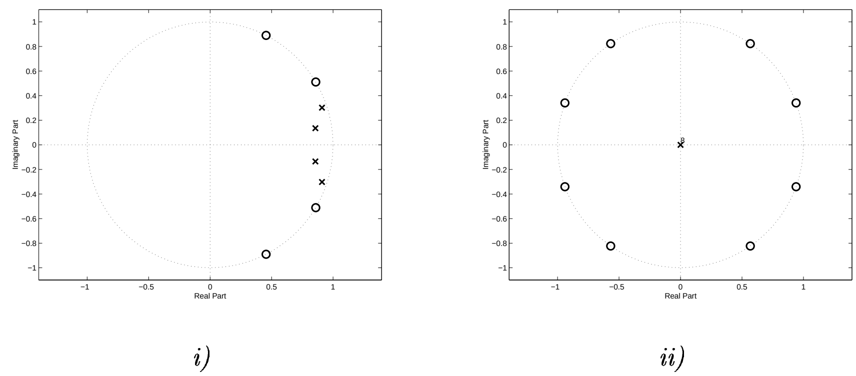

Figure 1: Zero-pole diagrams of Problem 2.

3. (6p) Figure 2 shows the flow diagram of a LTI system. The coefficient a is a finite constant.

   a) Derive the impulse response $h[n]$ of the filter.

   b) Determine the possible values of a so that the filter is stable.

   c) Let $x[n] = \cos(\frac{\pi}{2}n)\mu[n]$. Determine the coefficient a so that with the input $x[n]$ the output $y[n]$ of the filter is zero when $n > 1$.

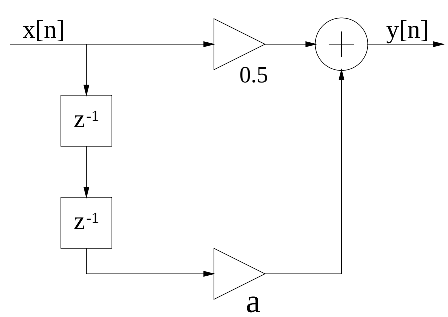

Figure 2: The flow diagram of Problem 3.

4. (6p) Consider an analog signal

$$x(t) = 2\cos(8\pi f_0 t) + 4\cos(20\pi f_0 t) + 5\cos(26\pi f_0 t) + 3\cos(32\pi f_0 t)\ ,$$

where $f_0 = 0.5$ kHz. The sampling frequency f_T in this problem is 6 kHz.

a) What is the length of the basic period of the signal $x(t)$?

b) Sketch the spectrum of the signal obtained by sampling $x(t)$ in the range $0\ldots3$ kHz.

c) Let us assume, that the interesting components of the spectrum are found at the frequency band $6\ldots9$ kHz. Suppose we want to store only the interesting components into a digital signal to the band $0\ldots3$ kHz for later inspection. How can this be done?

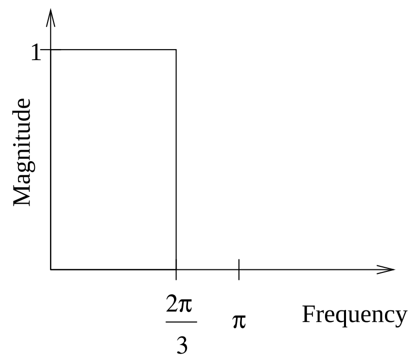

Figure 3: The ideal low-pass filter of Problem 5.

5. (6p)

a) Design a FIR low-pass filter based on Figure 3. Choose the order as 4 ($M = 2$). Use the truncated Fourier transform method (rectangular window).

b) Design a corresponding filter using the Hamming window function $w_h[n]$:

$$w_h[n] = 0.54 + 0.46\cos\left(\frac{2\pi n}{2M+1}\right)\ , \qquad -M \le n \le M$$

c) Explain how the frequency responses of the filters designed in a) and b) differ assuming that the orders of the filters are high enough (e.g. 50).

Hints:

Inverse Fourier transform:

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(j\omega)e^{j\omega n}d\omega$$

A useful intergral:

$$x[n] = \frac{1}{2\pi}\int_{-a}^{a} e^{j\omega n}d\omega = \frac{\sin(an)}{n\pi}$$

6. (6p) Consider a second-order digital IIR filter whose transfer function is

$$H(z) = \frac{1}{1 + 2a_1 z^{-1} + a_2 z^{-2}}$$

Sketch the possible locations of the poles (p_1, p_2) of the system, when the real coefficients a_1 and a_2 are quantized into three bits using the sign-magnitude representation. The numbers possible to represent are thus $-\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$. How does the quantization affect the realization of the filter in different frequencies?

Notice that the complex poles of a real coefficient filter are complex conjugates ($p_1 = re^{j\theta}$, $p_2 = p_1^* = re^{-j\theta}$).

7. (6p) What outputs of the system shown in Figure 5 need to be summed so that spectrum $X(j\omega)$ in Figure 4 of the input signal is transformed into the spectrum of the output signal $Y(j\omega)$? Sketch the spectrum of each output signal you use. In the system, there are thus up and downsamplers as well as ideal filters. The filters pass either one half of the frequency range (e.g. $[0, \frac{\pi}{2}]$) or one fourth (e.g. $[\frac{3\pi}{4}, \pi]$). Let us assume, that the signal power reduction caused by downsampling has been normalized in the output signals.

Hint: Begin the inspection of the outputs with the signals $y_1 - y_4$. After that, fill out the missing components of the spectrum with suitable ones from the signals $y_5 - y_{20}$.

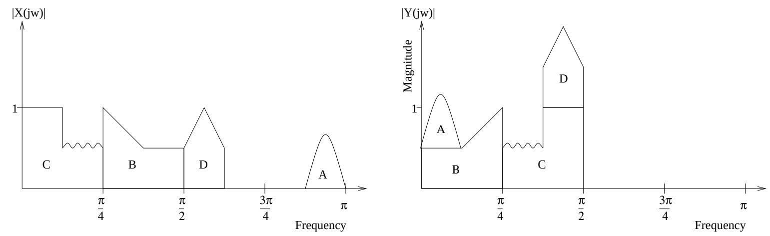

Figure 4: Spectra of the input and output signals of Problem 7.

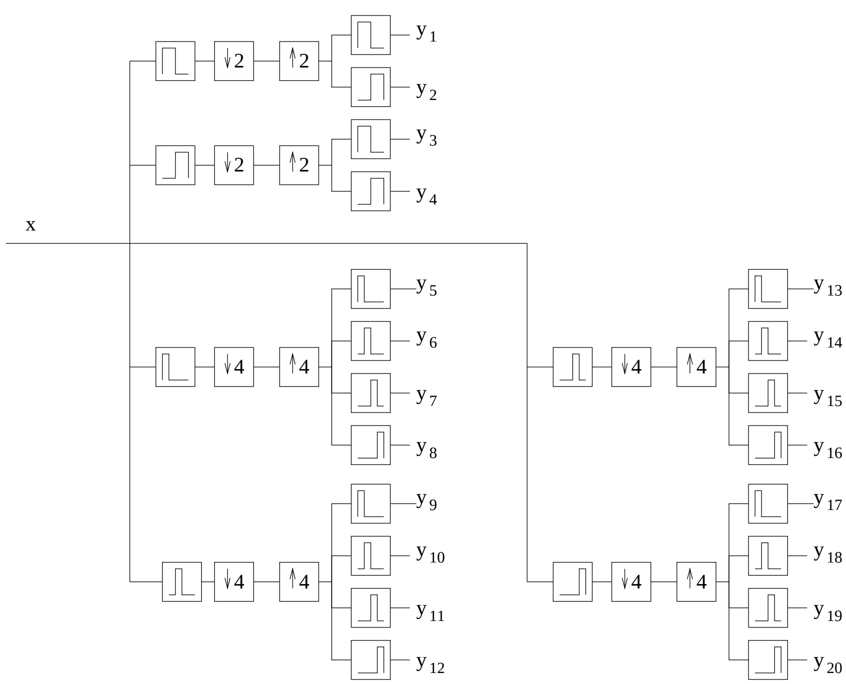

Figure 5: The system in Problem 7.