T-61.3010 Digital Signal Processing and Filtering

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The problems marked with [**Pxx**] are from the course exercise material (Spring 2009), where **Pxx** refers to the problem.

In the end of this session **you should know**: (a) what convolution (filtering) operation is, (b) how to compute discrete-time (de)convolution, (c) that we can observe frequency components in digital signals up to half of the sampling frequency (sampling theorem).

See before Problem 3 the aliasing demo `http://www.cis.hut.fi/Opinnot/T-61.3010/Demo/esim6.shtml` and / or using Matlab script `http://www.cis.hut.fi/Opinnot/T-61.3010/Demo/demosampling4.m`. Piip-piip.

1. [**P30**] Linear convolution of two sequences is defined as (*Mitra 2Ed Eq. 2.64a, p. 72 / 3Ed Eq. 2.73a, p. 79*)

$$y[n] = h[n] \circledast x[n] = x[n] \circledast h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

   a) Compute $x[n] \circledast h[n]$, when
   $x[n] = \delta[n] + \delta[n-1]$, and $h[n] = \delta[n] + \delta[n-1]$.
   What is the length of the convolution result?

   b) Compute $x_1[n] \circledast x_2[n]$, when
   $x_1[n] = \delta[n] + 5\delta[n-1]$, and $x_2[n] = -\delta[n-1] + 2\delta[n-2] - \delta[n-3] - 5\delta[n-4]$.
   What is the length of the convolution result? Where does the output sequence start?

   c) Compute $h[n] \circledast x[n]$, when
   $h[n] = 0.5^n \mu[n]$, and $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$.
   What is the length of the convolution result?

2. [**P32**] The impulse response $h_1[n]$ of a LTI system is known to be $h_1[n] = \mu[n] - \mu[n-2]$. It is connected in cascade (series) with another LTI system h_2 as shown in Figure 1.

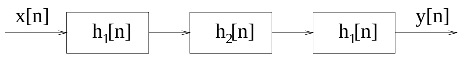

Figure 1: The cascade system of Problem 2.

Compute the impulse response $h_2[n]$, when it is known that the impulse response $h[n]$ of the whole system is shown in Table 1 below.

n	< 0	0	1	2	3	4	> 4
h[n]	0	1	5	9	7	2	0

Table 1: Impulse response of the cascade system in Problem 2.

3. [**P48**] Real analog signal $x(t)$, whose spectrum $|X(j\Omega)|$ is drawn in Figure 2, is sampled with sampling frequency $f_s = 8000$ Hz into a sequence $x[n]$.

   a) In the sampling process aliasing occurs. What would have been smallest sufficient sampling frequency, with which no aliasing would not happen?

   b) Analog signal $x(t)$ is 0.2 seconds long. How many samples are there in the sequence $x[n]$?

   c) Sketch the spectrum $|X(e^{j\omega})|$ of sampled sequence $x[n]$.

   d) Sequence $x[n]$ is filtered with a LTI system, whose pole-zero plot is shown in Figure 2. After that filtered sequence $y[n]$ is reconstructed (ideally) to continuous-time $y_r(t)$. Sketch the spectrum $|Y_r(j\Omega)|$ in range $f = [0 \ldots 20]$ kHz.

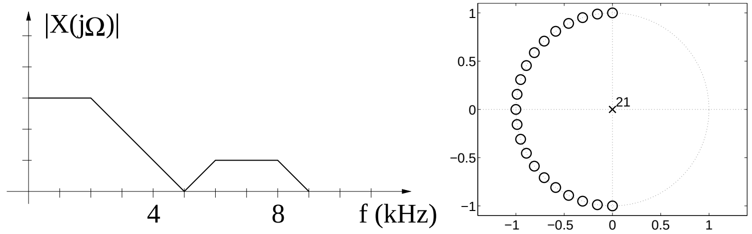

Figure 2: Problem 3: Spectrum left. Pole-zero plot right.