T-61.3010 Digital Signal Processing and Filtering

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The problems marked with [**Pxx**] are from the course exercise material (Spring 2009), where **Pxx** refers to the problem.

In the end of this session **you should know**: (a) how to define filter specifications and how to check if specifications are met, (b) how to design digital IIR filters using bilinear transform, (c) how to design digital FIR filters using window method.

1. [**P66**] Sketch the following specifications of a digital filter on paper. Which of the amplitude responses of the realizations in Figure 1 do fulfill the specifications?

   Specifications: Digital lowpass filter, sampling frequency f_T 8000 Hz, passband edge frequency f_p 1000 Hz, transition band 500 Hz (transition band is the band between passband and stopband edge frequencies!), maximum passband attenuation 3 dB, minimum stopband attenuation 40 dB.
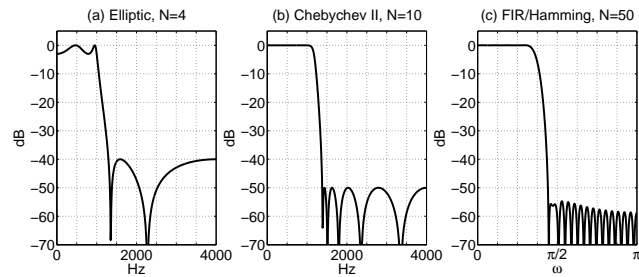

Figure 1: Three realizations in Problem 1: amplitude responses of (a) 4th order elliptic, (b) 10th order Chebychev II, (c) 50th order FIR using Hamming window.

2. [**P68**] Consider the following prototype analog Butterworth-type lowpass filter

$$H_{protoLP}(s) = \frac{1}{s+1}$$

   a) Form an analog first-order lowpass filter with cutoff frequency Ω_c by substituting $H(s) = H_{protoLP}(\frac{s}{\Omega_c})$. Draw the pole-zero plot in s-plane.

   b) Implement a discrete first-order lowpass filter $H_{Imp}(z)$, whose cutoff frequency (-3 dB) is at $f_c = 100$ Hz and sampling rate is $f_s = 1000$ Hz, applying the impulse-invariant method to $H(s)$. Draw the pole-zero plot of the filter $H_{Imp}(z)$.

   c) Implement a discrete first-order lowpass filter $H_{Bil}(z)$ with the same specifications applying the bilinear transform to $H(s)$. Prewarp the edge frequency. Draw the pole-zero plot of the filter $H_{Bil}(z)$.

3. [**P70**] Use windowed Fourier series method and design a FIR-type (causal) lowpass filter with cutoff frequency $3\pi/4$. Let the order of the filter be 4.

   See Figure 2, in left the amplitude response of the ideal lowpass filter $H(e^{j\omega})$ with cut-off frequency at $3\pi/4$. In right, the corresponding inverse transform of the desired ideal filter $h_d[n]$, which is sinc-function according to the transform pair rect(.) $\leftrightarrow$ sinc(.):

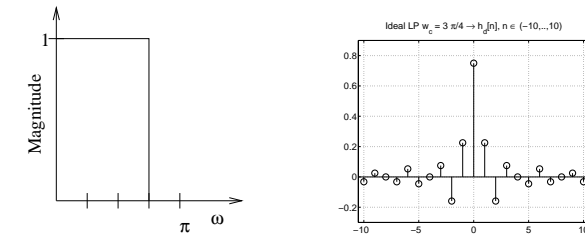$$h_d[n] = \{\ldots, -0.1592, 0.2251, \underline{0.75}, 0.2251, -0.1592, \ldots\}$$

Figure 2: Problem 3: (a) The amplitude response of the ideal lowpass filter, and (b) the corresponding impulse response $h[n]$ values. The cut-off frequency is at $\omega_c = 3\pi/4$.

   a) Use the rectangular window of length 5, see Figure 3(a). The window function is $w_r[n] = 1, -M \leq n \leq M, M = 2$

   b) Use the Hamming window of length 5, see Figure 3(b). The window function is

$$w_h[n] \quad = \quad 0.54 + 0.46\cos\left(\frac{2\pi n}{2M}\right) \quad , \quad -M \leq n \leq M, M = 2$$

   which results to $w_h[n] = \{0.08, 0.54, \underline{1}, 0.54, 0.08\}$

   c) Compare how the amplitude responses of the filters designed in (a) and (b) differ assuming that the window size is high enough (e.g. $M = 50$).
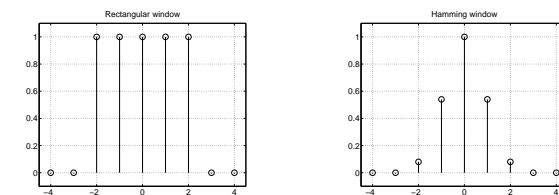

Figure 3: Problem 3: (a) rectangular window $w_r[n]$ of length 5, and (b) Hamming window $w_h[n]$ of length 5.