T-61.246 Digital Signal Processing and Filtering

Summer exam 21st June 2004.

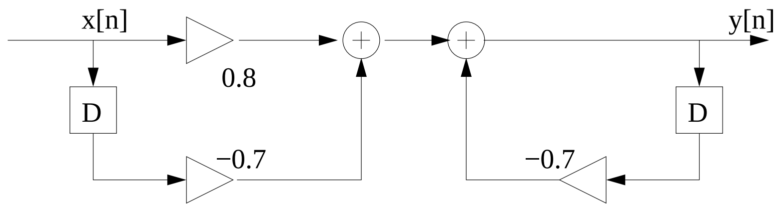
Equipment: pencil, (function) calculator, mathematical formulae. A Fourier transform table is delivered in the exam.

Write down some intermediate phases to your final answer.

1) (6p) Reply to statements, if they are true or false. Correct answer +1p, wrong -0.5p, no answer 0p.

   a) Equation $y[n] = (x[n])^2$ represents second order LTI-system.
   b) The fundamental period of the sequence $x[n] = \sin(0.2\pi n)$ is $N_0 = 100$.
   c) The convolution of the sequences $x[n] = \delta[n] + \delta[n+1]$ and $h[n] = \delta[n] - \delta[n-1]$ is $y[n] = x[n] \circledast h[n] = \delta[n] - \delta[n-2]$.
   d) The phase response $\angle H(e^{j\omega})$ of the filter $H(z) = 0.5 - 0.5z^{-1}$ is linear.
   e) The bad side of the bilinear method is aliasing, if the analog filter is not bandlimited.
   f) Using error-shaping structure the quantization noise can be modified and moved to a desired band.

2) (6p) Consider a discrete-time system, whose flow (block) diagram is shown below.



   a) What is the difference equation, i.e. what is $y[n] = \ldots$?
   b) Is the filter FIR or IIR? What is the order of the filter?
   c) Determine the frequency response transfer function $H(z) = Y(z)/X(z)$.
   d) Compute the poles and zeros, and sketch the pole-zero-diagram.
   e) Sketch the amplitude response $|H(e^{j\omega})|$. Is the filter of type lowpass, highpass, bandpass, bandstop or all-pass?

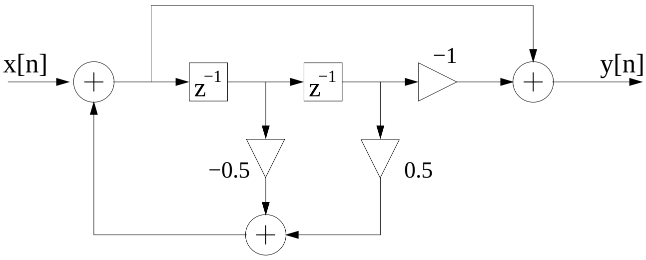
3) (6p) Consider a cosine signal $x(t) = \cos(2\pi \cdot f \cdot t)$, where the frequency is $f = 6 \cdot 10^5$ Hz. Signal is sampled using the sampling frequency $f_s = 44100$ Hz. The sampled sequence $x[n]$ is returned back ideally to continuous $x_r(t)$.

   a) What is the sampling period T_s?
   b) Which frequency is observed in the reconstructed signal $x_r(t)$?

4) (6p) **Reply to either A or B.**

4A) Digital filter $H(z)$ can be realized with different structures. What structures do exist and what differences are there?

4B) Consider a flow (block) diagram below.



a) What is the transfer function at its simpliest form?

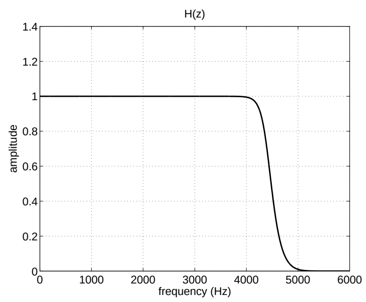
b) Compute the poles and zeros. Sketch the pole-zero-diagram. Is the realization in the figure canonic?

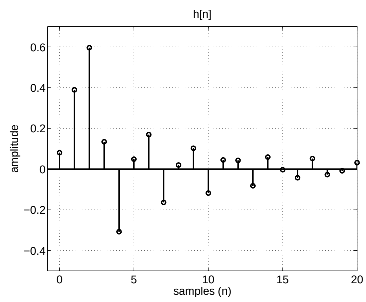
c) Determine from the pole-zero-diagram, if the filter is lowpass or highpass. Scale the filter with a constant K so that the maximum value of the filter is 1, i.e. $|K \cdot H_{max}(e^{j\omega_{max}})| = 1$.

5) (6p) **Reply to either A or B.**

5A) Design of digital FIR filter using window method.

5B) Consider a causal lowpass filter $H(z)$, whose passband ends at 4 kHz, stopband starts from 5 kHz and the sampling frequency is 12 kHz. The amplitude response is in the left figure and the start of the impulse response $h[n]$ in shown in right. Modify the filter so that it can handle DAT-recordings with the sampling frequency of 48 kHz.





a) Increase the sampling frequency with the factor $L = 4$. Draw the amplitude spectrum of the upsampled filtern $H(z^4)$ in range $0 \ldots 24$ kHz and the first ten values of the impulse response $h[n/4]$.

b) What has to be done so that the filter $H(z^4)$ works as a lowpass filter with the original cut-off frequencies?