T-61.3010 Digital Signal Processing and Filtering

Mid term exam 1, Sat 6.3.2010 at 10-13, main building.

You are allowed to do MTE1 only once either 7.3. or 13.3.

You are not allowed to use any calculators or math reference books. A list of formulas is delivered in the exam. A special form is delivered for Problem 1.

Return a special form and the other answer paper separately. Both ones have to have at least student number and name written on. Problem paper and the formulas you may keep.

Problem 3 is a course feedback which is open from Sat 6-March to Mon 22-March 2010.

1) (0-9 p) Multichoice statements. There are 1-4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

   Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 9 and the minimum 0.

   1.1 Consider a sequence $x[n] = A_1 \cos(\omega_1 n + \theta_1) + A_2 \cos(\omega_2 n + \theta_2) + A_3 \cos(\omega_3 n + \theta_3)$, where fundamental periods of each subsequence are $N_1 = 5$, $N_2 = 8$ and $N_3 = 10$, and A_i are non-zero. What can be said about periodicity of sequence $x[n]$?
   (A) Fundamental period N_0 exists if and only if all phases are zero: $\theta_1 = 0$, $\theta_2 = 0$, $\theta_3 = 0$
   (B) Fundamental period N_0 exists if and only if all coefficients A_i are equal
   (C) Fundamental period $N_0 = 40$
   (D) Fundamental period $N_0 = 400$

   1.2 Compute linear convolution $y[n] = h[n] \circledast x[n]$ of sequences $x[n] = \delta[n] + 2\delta[n-1] + \delta[n-2] = \{\underline{1},\ 2,\ 1\}$ and $h[n] = \delta[n] - 2\delta[n-1] = \{\underline{1},\ -2\}$. where underline shows the origin.
   (A) Length of $y[n]$ is 5
   (B) $y[n] = 0$, kun $n \leq 0$
   (C) $y[1] = -4$
   (D) $y[1] = 0$

   1.3 Two LTI systems $h_1[n]$ and $h_2[n]$ in cascade (serial) form the total impulse response $h[n]$ of the system. We know that $h_2[n] = \{1,\ \underline{2},\ -1\}$ and $h[n] = \{-2,\ -5,\ \underline{1},\ 3,\ -1\}$, where underline shows the origin. Hence, the unknown $h_1[n]$ is of form
   (A) $h_1[n] = a \cdot \delta[n+2] + b \cdot \delta[n+1] + c \cdot \delta[n] + d \cdot \delta[n-1] + e \cdot \delta[n-2]$
   (B) $h_1[n] = b \cdot \delta[n+1] + c \cdot \delta[n] + d \cdot \delta[n-1]$
   (C) $h_1[n] = d \cdot \delta[n-1] + e \cdot \delta[n-2] + f \cdot \delta[n-3]$
   (D) $h_1[n]$ is a causal filter
   where $\{a,\ b,\ c,\ d,\ e,\ f\} \in \mathbb{R}$ and non-zero.

   1.4 Impulse response of a LTI system is $h[n] = \sum_{k=0}^{\infty} (-1)^k \delta[n - 3k]$
   (A) The length of the impulse response is infinite
   (B) Filter is not stable
   (C) Filter is not causal
   (D) Corresponding difference equation is $y[n] = x[n] - y[n-3]$

   1.5 Based on properties of discrete systems, what can be said about the system in Figure 1?
   (A) It is a FIR filter
   (B) It is a linear and time-invariant system
   (C) It is a stable filter
   (D) It is a causal filter

   1.6 We know a band-limited spectrum $|X(j\Omega)|$ of an analog real-valued signal $x(t)$, see Figure 2(a). The signal is sampled with sampling frequency $f_T = 10000$ Hz.
   (A) The spectrum $|X(e^{j\omega})|$ of the sampled sequence in range $[0, f_T/2]$ is in Figure 3(a). (y-axis values propotional.)
   (A) The spectrum $|X(e^{j\omega})|$ of the sampled sequence in range $[0, f_T/2]$ is in Figure 3(b). (y-axis values propotional.)
   (C) The obtained sequence $x[n]$ is a sinusoidal of form $x[n] = \cos(\omega_0 n + \theta)$, where $\omega_0 = 2\pi(f_0/f_T)$ is normalized fundamental angular frequency
   (D) All those frequency components, whose period T_i is shorter than $2/f_T$ seconds, fold (alias) to lower frequency in range $[0, f_T/2]$ Hz of the digital spectrum $|X(e^{j\omega})|$

1.7 The transfer function of the filter is

$$H(z) = \frac{1+(0.2-0.4j)z^{-1}}{1-0.8z^{-1}} \cdot \frac{1+(0.2+0.4j)z^{-1}}{1+0.9z^{-1}} \cdot \frac{1}{1-0.7z^{-1}}, \qquad |z| > 0.9$$

(A) The pole-zero diagram is in Figure 4(a)
(B) The magnitude response is in Figure 4(b)
(C) The order of the filter is 5
(D) The filter has a linear phase response

1.8 Discrete Fourier transform (DFT) of a sequence $x_1[n] = \{\underline{1}, 2, 2, 1\}$ is

$$X_1[k] = \sum_{n=0}^{3} x_1[n] W_N^{nk} = \{\underline{6},\ -1-j,\ 0,\ -1+j\}$$

and correspondingly for $x_2[n]$ there are $x_2[n] = \{\underline{1},\ 1,\ 0,\ 0\}$ and $X_2[k] = \{\underline{2},\ 1-j,\ 0,\ 1+j\}$. Compute DFT $X_3[k]$ of a sequence $x_3[n] = x_1[n] + 2x_2[n]$. (DFT can be found in formula table.)

	$k =$	0	1	2	3
(A)	$X_3[k] =$	10	$1-3j$	$2j$	$2+2j$
(B)	$X_3[k] =$	10	$2-2j$	3	$2+2j$
(C)	$X_3[k] =$	10	$1-3j$	0	$1+3j$
(D)	$X_3[k] =$	10	$3-j$	1	$-1+3j$

1.9 Consider a LTI filter whose transfer function is

$$H(z) = 1 + z^{-8}$$

(A) It is a comb filter
(B) Zeros of the filter are at $d_1 = +j$ and $d_2 = -j$
(C) The length of impulse response $h[n]$ is 8
(D) The group delay is $\tau(\omega) = 8$

1.10 Read a file into Matlab with command `[x, fT] = wavread('kiisseli.wav');`. There is an audio signal $x[n]$ with sampling frequency $f_T = 22050$ Hz. It is fed into a LTI system with impulse response

$$h[n] = \sum_{k=0}^{9} \frac{10-k}{50} \cdot \delta[n-k]$$

and an output $y[n]$ is obtained with command `y = conv(h, x);`

(A) The filter can filter out noise at 50 Hz
(B) The filter produces audible "echo effect"
(C) It is a linear-phase filter, and therefore there is no phase distortion
(D) None of above holds

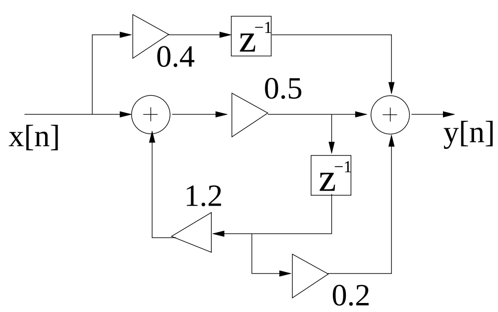

Figure 1: Statement 1.5: Flow diagram of the filter.

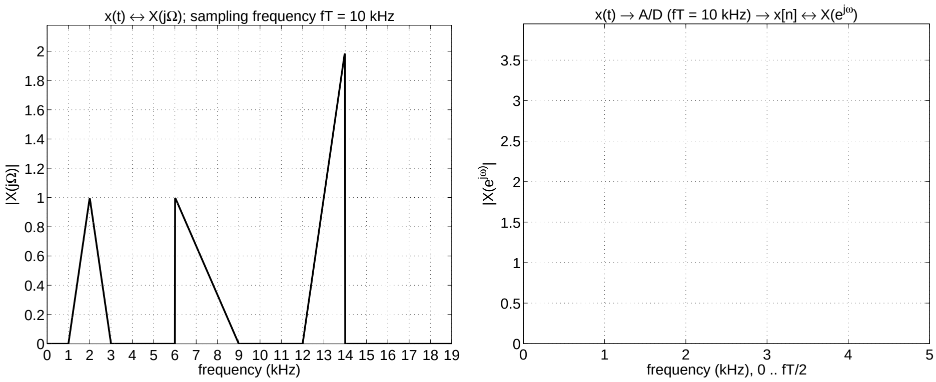

Figure 2: Statement 1.6: (a) Spectrum $|X(j\Omega)|$ of analog signal $x(t)$, (b) empty axis $f \in [0, f_T/2]$ for sketching.

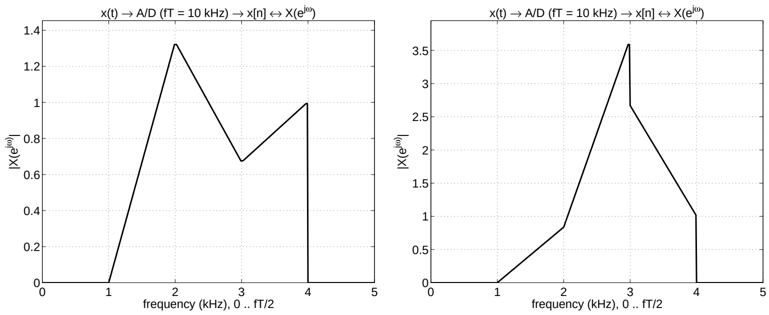

Figure 3: Statement 1.6: (a) option **(A)** , (b) option **(B)** .

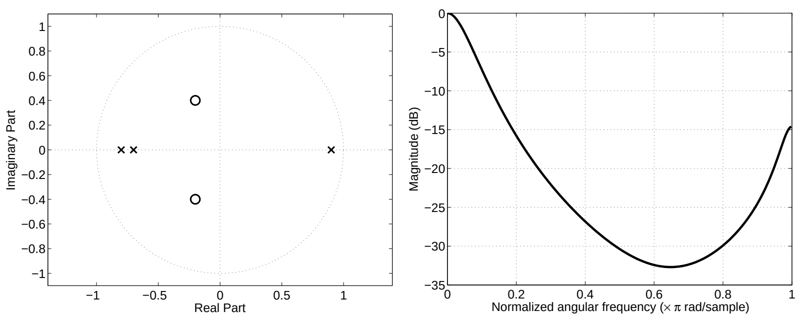

Figure 4: Statement 1.7: (a) option **(A)** , (b) option **(B)** .

2) (6 p) Consider a discrete-time linear and time-invariant system, whose impulse response is

$$h[n] = 4 \cdot (-0.8)^n \mu[n] - 3 \cdot (-0.6)^n \mu[n]$$

Examine the filter and its behavior with tools given in the course. Write down the facts as clearly as possible.

3) (1 p) Course feedback. Questionnaire `http://www.cis.hut.fi/Opinnot/T-61.3010/VK1_K2010/kyselyVK1_en.shtml` is open till 22-Mar 2010.