T-61.3010 Digital Signal Processing and Filtering

Mid term exam 1, Mon 10.3.2008 at 13-16. Hall M (non-Finnish).

You are allowed to do MTE1 only once either 7.3. or 10.3.

You are not allowed to use any calculators or math reference books. A list of formulas is delivered in the exam. A special form is delivered for Problem 1.

Return a special form and the other answer paper separately. Both ones have to have at least student number and name written on. Problem paper and the formulas you may keep.

Write down all necessary intermediate steps in Problem 2. Problem 3 is a course feedback which is open from Wed 12-March to Wed 19-March 2008.

1) (10 x 1p, max 8 p) Multichoice statements. There are 1-4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

   Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 8 and the minimum 0.

   1.1 Compute partial fraction expansion for function $f(x) = \frac{x+1.8}{x^2-0.81}$
   - **(A)** $f(x) = \frac{-(1/6)j}{x-0.9j} + \frac{1+(1/6)j}{x+0.9j}$
   - **(B)** $f(x) = \frac{0.5x-0.3}{x-0.9j} - \frac{0.5x-0.3}{x+0.9j}$
   - **(C)** $f(x) = \frac{1.5}{x-0.9} - \frac{0.5}{x+0.9}$
   - **(D)** $f(x) = \frac{0.5x+2}{x-0.9} - \frac{0.5x}{x+0.9} - 1$

   1.2 It is possible to analyze if the sequence $x[n] = \cos((2\pi/9)n^3)$ is periodic or not by substitution $n \leftarrow (n+N)$. In this way we get $x_N[n] = \cos((2\pi/9)n^3 + 2\pi(\frac{3n^2N+3nN^2+N^3}{9}))$, from which one can see that $x[n]$
   - **(A)** is not periodic
   - **(B)** is periodic but its fundamental period N_0 is complex-valued
   - **(C)** is periodic with fundamental period $N_0 = 3$
   - **(D)** None of above is true

   1.3 Compute the linear convolution $y[n] = h[n] \circledast x[n]$ of impulse response $h[n] = \delta[n-1] + 4\delta[n-2] - 2\delta[n-3]$ and input $x[n] = \delta[n+2] - 0.84\delta[n+1] - 0.231\delta[n] + 0.72\delta[n-2] - \delta[n-3] - \delta[n-4]$
   - **(A)** $y[n] = \{1,\ \underline{3.16},\ -5.591,\ 0.756,\ 1.182,\ 1.88,\ -6.44,\ -2,\ 2\}$
   - **(B)** $y[n] = \{\underline{0},\ 1,\ 3.16,\ -5.591,\ 1.182,\ 0.756,\ 1.88,\ -6.44,\ 2,\ -2\}$
   - **(C)** $y[n] = \{\underline{0},\ 0,\ 2.88,\ 2\}$
   - **(D)** $y[n] = \{\underline{0},\ 0,\ 0,\ 1,\ 3.16,\ -5.591,\ 0.476,\ 3.182,\ -3,\ -2\}$

   1.4 In Figure 1 there is the spectrum $|X(j\Omega)|$ of analog signal. Now that the signal is sampled with sampling frequency $f_T = 12$ kHz, the spectrum $|X(e^{j\omega})|$ of discrete signal (ignore y-axis scaling term) is
   - **(A)** in Figure 2(a)
   - **(B)** in Figure 2(b)
   - **(C)** in Figure 2(c)
   - **(D)** in Figure 2(d)

   1.5 Analog signal is sampled with sampling period $T = 1/f_T = 4$ ms. The digital sequence is of form $x[n] = \cos(2\pi(80/f_T)n)$. Which of the following analog signals could have been the source?
   - **(A)** $x(t) = \cos(2\pi \cdot 320 \cdot t)$
   - **(B)** $x(t) = \cos(2\pi \cdot 420 \cdot t)$
   - **(C)** $x(t) = \cos(2\pi \cdot 540 \cdot t)$
   - **(D)** $x(t) = \cos(2\pi \cdot 680 \cdot t)$

   1.6 A lowpass filter $H_{LP}(e^{j\omega})$ can be easily changed to a highpass filter $H_{HP}(e^{j\omega})$ by shifting the frequency axis by π $H_{HP}(e^{j\omega}) = H_{LP}(e^{j(\omega-\pi)})$ according to properties of discrete-time Fourier-transform (DTFT). Hence, from lowpass filter $H_L(z) = 0.5 + 2.5z^{-1} + 2.5z^{-2} + 0.5z^{-3}$ one can obtain highpass filter $H_H(z)$ with impulse response
   - **(A)** $h_h[n] = -0.5\delta[n] - 2.5\delta[n-1] - 2.5\delta[n-2] - 0.5\delta[n-3]$
   - **(B)** $h_h[n] = 0.5\delta[n] - 2.5\delta[n-1] + 2.5\delta[n-2] - 0.5\delta[n-3]$
   - **(C)** $h_h[n] = 0.5e^{j\pi}\delta[n] + 2.5e^{j\pi}\delta[n-1] + 2.5e^{j\pi}\delta[n-2] + 0.5e^{j\pi}\delta[n-3]$
   - **(D)** $h_h[n] = 1 - |0.5n + 2.5(n-1) + 2.5(n-2) + 0.5(n-3)|$

1.7 Consider a causal and stable LTI filter $H(z) = \frac{1+1.8z^{-1}}{1-0.81z^{-2}}$.

(A) Impulse response in a non-recursive form is $h[n] = (0.81)^{2n}\mu[n] - 0.6 \cdot (0.81)^{2n-1}\mu[n-1]$

(B) Impulse response in a non-recursive form is $h[n] = 1.5 \cdot (-0.9)^n\mu[n] - 0.5 \cdot 0.9^n\mu[n]$

(C) Impulssivaste ei-rekursiivisessa muodossa on $h[n] = 5 \cdot (-0.9)^{-n-1}\mu[-n-1] + 4 \cdot 0.9^{-n-1}\mu[-n-1]$

(D) Value of impulse response at moment $n = 2008$ is $h[2008] = 9^{2008}/10^{2008}$

1.8 Zeros of causal and stable LTI filter are at $d_1 = 1$, $d_2 = j$, $d_3 = -1$, $d_4 = -j$ and poles at $p_1 = 0.8e^{j(\pi/4)}$, $p_2 = 0.8e^{j(3\pi/4)}$, $p_3 = 0.8e^{j(-3\pi/4)}$, and $p_4 = 0.8e^{j(-\pi/4)}$.

(A) The order of the filter is 8

(B) Impulse response is of form $h[n] = K \cdot \big(\delta[n] - \delta[n-4]\big) + 0.8e^{j\pi/4} \cdot \big(\delta[n] + j\delta[n-1] - \delta[n-2] - j\delta[n-3]\big)$

(C) Scaled amplitude response is in Figure 3, where x-axis is $\omega = [0 \ldots 1 \cdot \pi]$

(D) None of above is true

1.9 Consider a LTI filter $y[n] = \frac{1}{2008} \cdot (x[n] - x[n-2008]) + y[n-1]$.

(A) Filter is not stable

(B) Filter is not causal

(C) Filter is a recursive version of MA-2008-filter ("moving average")

(D) Filter has a pole and a zero at $z = -1$ which cancel each other

1.10 Block diagram of the filter in Figure 4 is analyzed with Matlab. Correct commands are

(A)

```
num = 0.2 * [1 0 2 0 -1];
den = 0.005 * [-100 -50 25 -1];
figure(42);
plot(freqz(num, den));
figure(43);
plot(zplane(num, den));
```

(B)

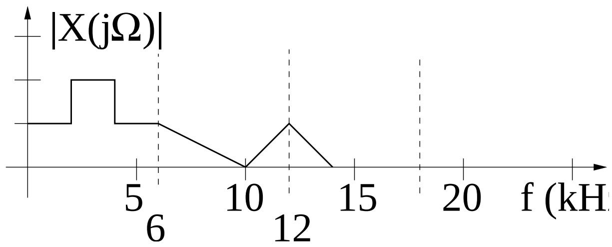
```
B = [0.2 0.4 -0.2];
A = [0.5 0.25 -0.125 0.005];
figure(13);
freqz(B[1,3,5], A[2,3,4,5]);
figure(14);
zplane(B[1,3,5], A[2,3,4,5]);
```

(C)

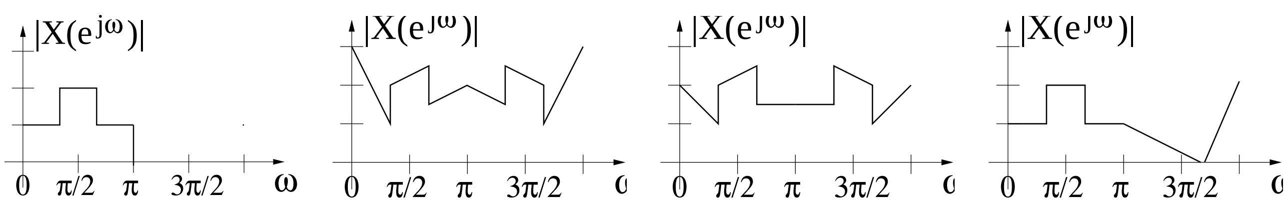
```
B = [0.2 0 0.4 0 -0.2];
A = [1 -0.5 -0.25 0.125 -0.005];
figure(42);
abs(plot(B, A, 'g'));
hold on;
angle(plot(B, A, 'k'));
```

(D)

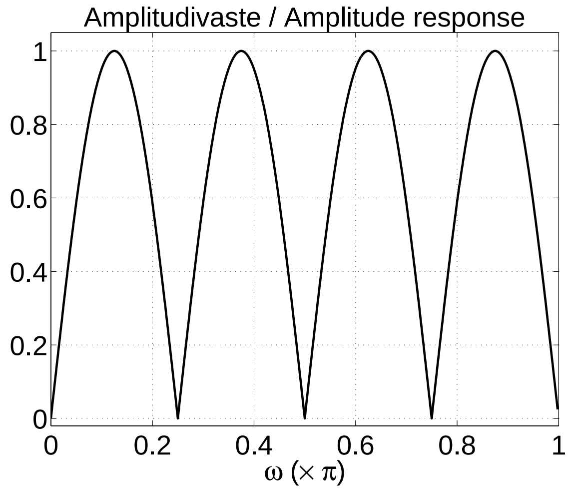
```
B = [.2 0 .4 0 -.2];
A = [1 .5 .25 -.125 .005];
figure(6); clf;
freqz(B, A, 512, 22050);
figure(7); clf;
zplane(B, A);
```



Kuva 1: Multichoice statement 1.4, analog spectrum.



Kuva 2: Multichoice statement 1.4, answers **(A)** , **(B)** , **(C)** ja **(D)** .



Kuva 3: Multichoice statement 1.8, option **(C)** .

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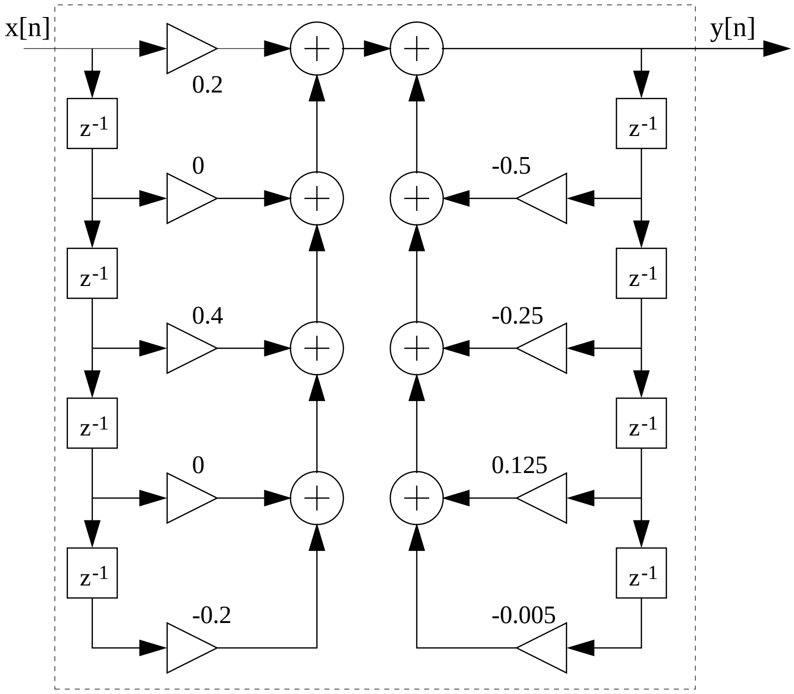

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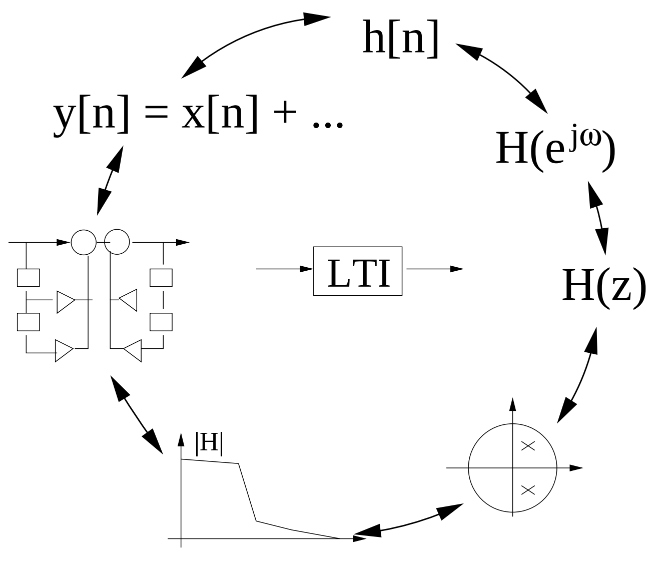
Kuva 4: Multichoice statement 1.10, block diagram.

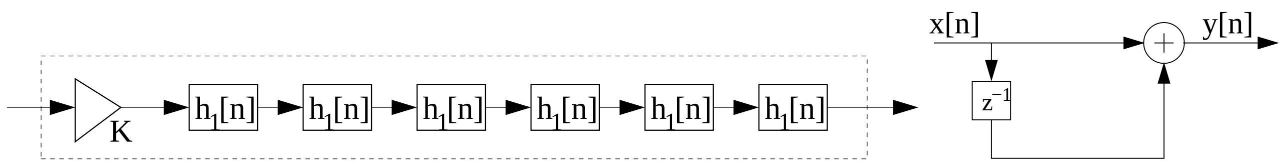
2) (6p) In the beginning of this course we have analyzed digital signals and linear and time-invariant (LTI) filters. The analysis is both in time and frequency domain as shown in Figure 5.



Kuva 5: Problem 2. A big picture of LTI analysis. The filter can be analyzed in time- and frequencydomain in various ways.

Examine now a filter $h[n]$ given in Figure 6(a) with the scaling factor K and six similar LTI filters $h_1[n]$ in cascade (series). A single, linear-phase FIR filter $h_1[n]$ is given in Figure 6(b).

a) Analyze the filter $h_1[n]$ in frequency-domain as taught in this course.
b) Write down the frequency response $H(e^{j\omega}) = \ldots$ of the total filter so that the maximum of $|H(e^{j\omega})|$ is one.
c) Filter $h[n]$ is also linear-phase filter. Explain or show by computing.
d) When a periodic sequence is fed to a FIR filter, the output changes periodic ("steady-state response") after awhile ("transient response")
   If now in the output of filter $h[n]$ ("steady-state") there is a sequence $y[n] = 0.4 \cdot \cos((\pi/2)n + \pi/8)$, what has been the input sequence $x[n]$?



Kuva 6: Problem 2: (a) Total filter $h[n]$, with gain K. (b) A single filter $h_1[n]$.

3) Reply to course feedback questionnaire, which is open from Wed 12-March to Wed 19-March 2008. Instructions are sent by email. This is part of mid term exams and worth of +1 point.