Tik-61.145 Principles of digital signal processing

Exam, January 14, 1997

1. (6p) What are the outputs $y_1(n)$ and $y_2(n)$ of the system in Figure 1 at moment $n > 0$, when at $n = 0$ the upper register is preset to value A, and lower to 0 (*hint*: add the system an input branch, solve the transfer functions, and study the scaled impulse response)?

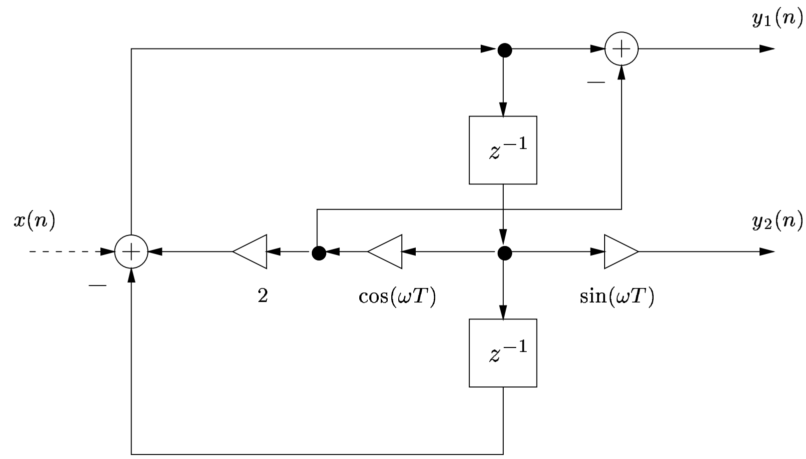

Figure 1: Flow diagram.

2. (6p) Consider the following two FIR-systems with impulse responses

$$\begin{aligned} h_1(n) &= \delta(n) + 3\delta(n-1) + 3\delta(n-2) + \delta(n-3) \\ h_2(n) &= \delta(n) - 2\delta(n-1) + \delta(n-2). \end{aligned}$$

   (a) Determine the impulse response corresponding to the composite cascade of the two systems. Sketch the impulse response.

   (b) Determine the frequency response of the composite cascade. Calculate amplitude and phase responses (in the simplest possible form) and sketch them. How does the phase of the composite cascade behave?

   (c) Determine the step response of the composite cascade. How does the response behave, when n is big? Why?

3. (6p) Consider an FIR filter having the transfer function

$$H(z) = 1 + 2.5z^{-1} + z^{-2}.$$

   (a) Draw the pole-zero diagram, and sketch $|H(e^{j\omega T})|$, the magnitude response of the filter. What kind of filter is this?

   (b) Draw the pole-zero diagram, and sketch the magnitude response of the same filter, when each delay of the original filter is replaced by four delays. What kind of filter we now have?

4. (6p) Sketch roughly the magnitude responses of Chebysev I and elliptic type IIR digital filters on the interval $[0 \ldots \pi]$, when the following specifications have been given. It is assumed that the filters are designed in the continuous domain and they have been digitized using bilinear transform.

   (a) 5rd order lowpass filter, whose
      * passband ends at frequency $\frac{\pi}{3}$ and
      * stopband begins at frequency $\frac{2\pi}{3}$.

   (b) 4nd order highpass filter, whose
      * stopband ends at frequency $\frac{\pi}{4}$ and
      * passband begins at frequency $\frac{\pi}{2}$.

   (c) 6th order bandstop filter, whose
      * 1st passband ends at frequency $\frac{\pi}{4}$,
      * stopband begins at frequency $\frac{\pi}{3}$,
      * stopband ends at frequency $\frac{2\pi}{3}$ and
      * 2nd passband begins at frequency $\frac{3\pi}{4}$.

5. (6p) Consider an analog filter whose s-plane transfer function is

$$H_c(s) = \frac{1}{s+1}.$$

   (a) Determine $H(z)$, the transfer function of the corresponding digital filter, when impulse-invariant transform is used in filter digitalization. (The impulse response of the continuous-time filter is $h_c(t) = e^{-t}$, when $t \geq 0$, 0 otherwise.)

   (b) Determine $H(z)$, the transfer function of the corresponding digital filter, when bilinear transform is used in filter digitalization (transformation formula: $s = \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$). It is assumed, that the frequency distortions have been taken into account in the design phase of the filter, and they do not need to be compensated.

   (c) Sketch the magnitude responses of both digital filters obtained using methods above when $T = 1$. How do the two methods differ (in general)?