T-61.246 Digital Signal Processing and Filtering

2nd mid term exam / final exam 13th Dec 2004 at 9-12. Halls A and B.
If you are doing 2nd MTE, reply to problems 3, 4, 5, 6.
If you are doing final exam, reply to problems 1, 2, 4, 5, 6.
Write down, if you are doing 2nd MTE or final exam.
You may use a (graphical) calculator. You must clear all extra memory in your calculator. There is an additional formulae table given in the exam, but you can also use a math reference book of your own. **Write down all necessary steps which lead to the results.**

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PLEASE, GIVE FEEDBACK IN WEB
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The link can be found also from the course web page.

1. (6p, final exam)

   a) (2p) What is the fundamental period N_0 of the sequence $x[n] = e^{j(\pi/4)n} + \cos((\pi/3)n)$?

   b) (2p) Sketch the amplitude response $|H(e^{j\omega})|$ of the filter $y[n] = x[n] - 2x[n-1] + x[n-2]$.

   c) (2p) Sketch the pole-zero-diagram of the filter

   $$H(z) = \frac{1 - 0.2z^{-1}}{1 + 0.64z^{-2}}$$

2. (6p, final exam) The input sequence $x[n]$ to a causal LTI system produces output $y[n]$. Known values of $x[n]$ and $y[n]$, and the parametrized impulse response $h[n]$ are as follows:

$$\begin{aligned} x[n] &= \delta[n] + 2\delta[n-1] - \delta[n-2] \\ y[n] &= -\delta[n-1] - \delta[n-2] + 5\delta[n-3] + 5\delta[n-4] + 4\delta[n-5] + 4\delta[n-6] + 4\delta[n-7] + \ldots \\ h[n] &= \begin{cases} a, & \text{when } n < 0 \\ b, & \text{when } n = 0 \\ c, & \text{when } n = 1 \\ d, & \text{when } n = 2 \\ e, & \text{when } n > 2 \end{cases} \end{aligned}$$

   a) (4p) Determine the unknown constants of the impulse response $h[n]$.

   b) (1p) Is the filter FIR or IIR? Explain.

   c) (1p) Is the filter stable? Explain.

3. (6p, MTE2) Are the following statements true (T) or false (F)? A right answer gives +1p, no answer 0 p, and a wrong answer –0.5p. Answer to as many statements as you want. You do not need to explain. The total amount of points is 0-6p.

   a) One possible polyphase realization of a FIR filter $H(z) = 1 - 0.4z^{-1} - 0.4z^{-2} + z^{-3}$ is $H(z) = F_0(z^2) + F_1(z^2)$, where $F_0(z) = 1 - 0.4z^{-1}$ ja $F_1(z) = -0.4 + z^{-1}$.
   b) Scaling of the filter is used to suppress the signal in order to reject overflows, and at the same time signal-to-noise ratio (SNR) is improved.
   c) The order of the elliptic IIR lowpass filter in Figure 1(a) is 2.
   d) Matlab code `plot(n, x)` produces a curve in Figure 1(b), when `n` refers to indices $n = 0 \ldots 4$ and `x` refers to sequence $x[n] = \{\underline{1}, 3, 2, 5, 4\}$.
   e) CD-quality audio has the interval (period) between each digital sample appr. 0.0227 ms.
   f) The order of the filter in Figure 1(c) is 4.
   g) Upsample a cosine sequence of 2000 Hertz so that the original sampling frequency 8 kHz is increased by double, that is with factor $L = 2$. Statement: The frequency of the downsampled signal is 4 kHz.
   h) A cascade (series) system of second-order systems is more sensitive to quantization of coefficients than the corresponding direct form structure.

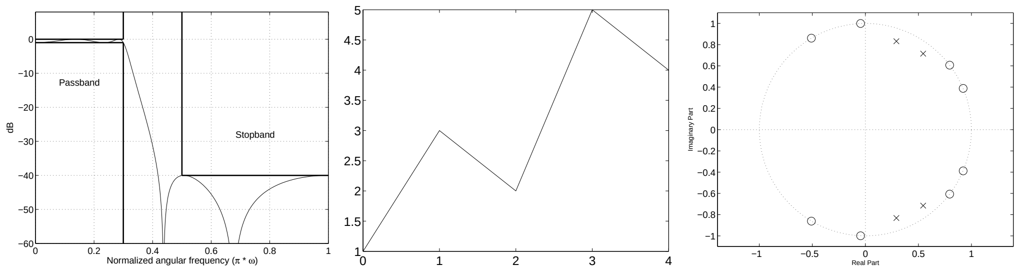

Figure 1: Figure (a), (b), and (c) for Problem 3.

4. (6p, MTE2, final exam) There are two second-order IIR filters in Figure 2. Consider only the complex poles of the filters, when the real coefficients a and b are quantized into three bits using sign-magnitude representation. The numbers possible are thus $\{-0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75\}$.

   Draw the positions of all possible complex poles of each filter, and compare them. Notice that the complex poles of a real-coefficient filter are complex conjugates ($p_1 = re^{j\omega}$, $p_2 = p_1^* = re^{-j\omega}$) and $1 + d_1z^{-1} + d_2z^{-2} = (1 - p_1z^{-1})(1 - p_2z^{-1})$.

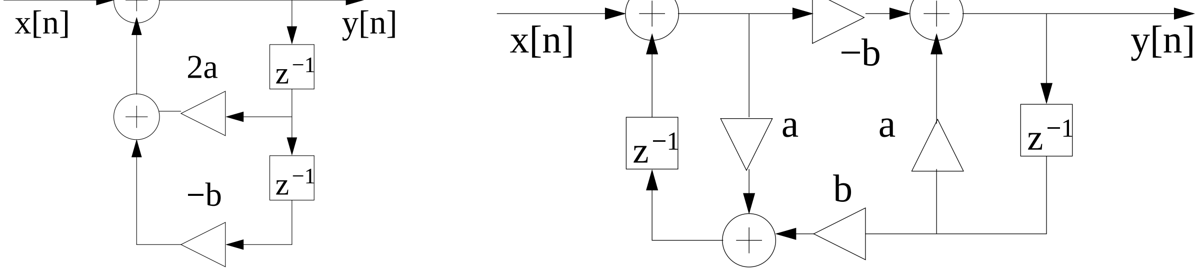

Figure 2: The filters of Problem 4.

5. (6p, MTE2, final exam) Consider a signal $x[n]$, whose amplitude spectrum is in Figure 3 in range $0 \dots \pi$. Only the frequency band $0 \dots \pi/5$ is needed for the signal. The sampling frequency is 16000 Hz and it will be increased digitally up to 28000 Hz.

a) Explain briefly which components and in which order are needed in this system, which changes sampling frequency.

b) Determine the passband and stopband cut-off frequencies of the required lowpass filter so that the filter has as low order as possible. Examine the procedure in frequency domain and show all steps.

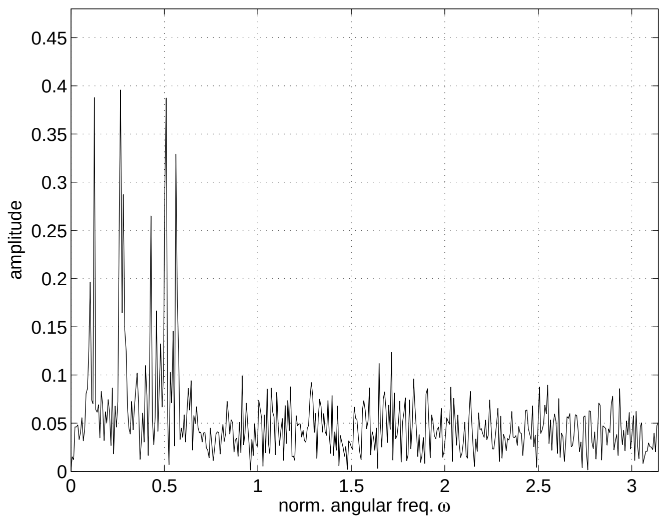

Figure 3: The spectrum of Problem 5.

6. (6p, MTE2, final exam) **Choose either A or B.**

6A. Essay: FFT-algorithms, especially "Decimation-in-Time" and "Decimation-in-Frequency". You do not have to derive formulas.

6B. Consider an analog transfer function $H_a(s) = (s+a)/[(s+a)^2+b^2]$, where coefficients a and b are real-valued. The pole-zero-plot of the filter (in s-plane) and amplitude response are as shown in Figure 4.

NOTE! You do not have to compute any z-plane transfer functions, or corresponding. Only sketching of figures is enough.

a) Sketch the amplitude response of a digital filter via impulse invariant method.

b) Sketch the amplitude response of a digital filter via bilinear transform.

c) Explain briefly, how the methods in a) and b) differ from each other.

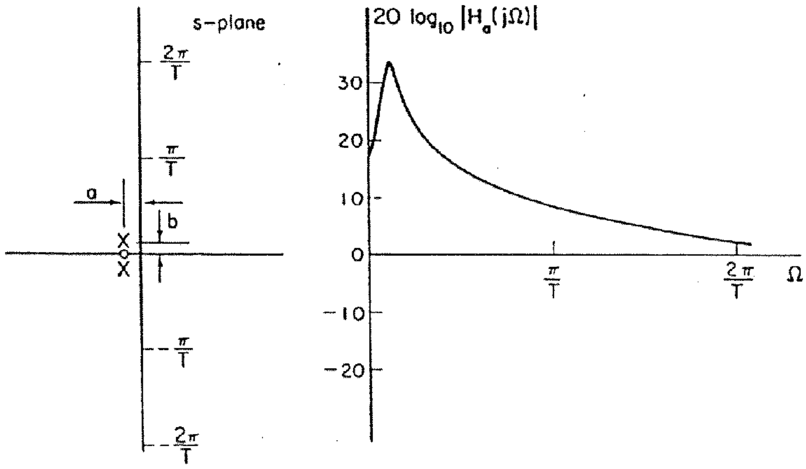

Figure 4: Problem 6B: analog s-plane pole-zero-plot in left, and the amplitude response $|H(j\Omega)|$ in right. $\Omega = 2\pi f$ (rad/s), $\omega = 2\pi(\Omega/\Omega_T)$ (rad), where Ω_T angular sampling frequency.