T-61.5030 Advanced course in neural computing Karhunen, Peltonen

Exercise 7, Nov. 9, 2006

1. Consider a random input vector $\mathbf{X}$ made up of two component vectors, $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2]^T$. Assume that you would like to represent the central dependencies between $\mathbf{X}_1$ and $\mathbf{X}_2$ with a simple model. How would you do that? (Hint: Consider linear combinations.)

2. Assume a mixture of time-delayed (but not convolved) sources with known delays D_{ij}:

$$x_i(t) = \sum_{j=1}^{N} a_{ij} s_j(t - D_{ij}),$$

   (a) Show that by Fourier transform, this can be reduced to the instantaneous mixture model $\mathbf{X} = \mathbf{AS}$.

   (b) From this mixing matrix $\mathbf{A}$, how do you solve the original mixing coefficients a_{ij}?

3. Prove the following properties of kurtosis by using the definition of kurtosis: If x_1 and x_2 are independent random variables, then

$$kurt(x_1 + x_2) = kurt(x_1) + kurt(x_2)$$

$$kurt(\alpha x_1) = \alpha^4 kurt(x_1)$$

   where α is a scalar.

4. Show that if $\mathbf{x}$ is the observed vector and

$$\mathbf{x}_2 = \mathbf{E}\mathbf{D}^{-1/2}\mathbf{E}^T\mathbf{x}$$

   Then $\mathbf{x}_2$ is white, where $\mathbf{E}$ is the orthogonal matrix of eigenvectors of $E[\mathbf{x}\mathbf{x}^T]$ and $\mathbf{D}$ is the diagonal matrix of its eigenvalues.

5. Consider the following distribution:

$$g(x) = \frac{b}{4}\{\exp(-b|x-a|) + \exp(-b|x+a|)\}$$

   (a) Using the general expression for the n^{th} order moment of a distribution $p(x)$, which has infinite support, $m_n = \int_{-\infty}^{\infty} p(x)x^n dx$ show that the kurtosis of g(x) is defined by the expression

$$\frac{12 - 2a^4b^4}{4 + 4a^2b^2 + a^4b^4}$$

   (b) Study the values of kurtosis for this distribution when the parameters a and b take on a range of values.

      i. When is the value of the kurtosis negative?

      ii. When is the value of the kurtosis positive?

      iii. When is the value of the kurtosis zero?

   (c) Is this distribution Sub- or Super-Gaussian? Discuss.