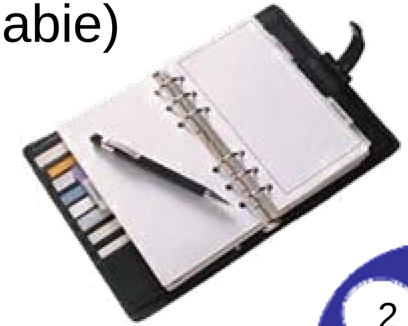
Deconvolution, Deblurring and Restoration

Igor Lissandron, 18.04.07

Agenda

- Introduction to image restoration
- Linear Space-Invariant restoration filters
  - Inverse filtering
  - Power Spectrum Equalization
  - Wiener filter
  - Constrained least-squared restoration
  - Metz filter
- Deblurring
  - Blind Deblurring
  - Iterative Blind Deblurring (Method of Rabie)



Agenda / 2

- Deconvolution
  - Homomorphic Deconvolution
- Space variant restoration
  - Sectioned image restoration
  - Adaptive-neighborhood deblurring
  - Kalman filter
- Restoration of nuclear medicine images
- SPECT images of the brain (example)

The Big Picture

Images quality improvement

- **Enhancement techniques**
  Better looking images satisfying some <u>subjective</u> criteria
  → The processed image may not be closer to the true image

- **Restoration**
  Try to find the best possible estimate of the original (and unknown) image following <u>objective</u> criteria
  → Possibility to exploit all the additional knowledge about the original image, imposing constraints to limit the solution

Introduction

Try to remove or minimize some known degradations in an image.

In order to do this, we should have:

- *Precise information about the degrading phenomenon*
- *Analysis of the system that produced the degraded image*

Typical items of information required:

→ *Models of the impulse response of the degrading filter*
→ *PSD of the original image*
→ *PSD of the noise*

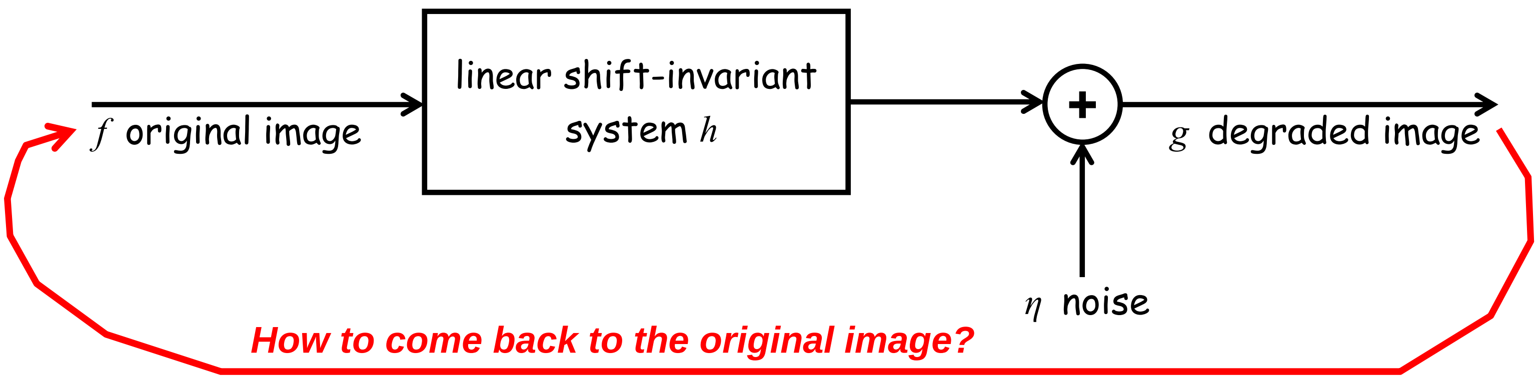
Image Restoration

Linear Space-Invariant restoration filters

If we assume that the degrading phenomenon is linear and shift-invariant, the simplest model of image degradation is

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v) \cdot F(u,v) + \eta(u,v)$$



Inverse filtering

Let's consider the degradation model expressed as

$$g = h \cdot f$$

We want to estimate f knowing g and h

1. Let's consider an approximation $\tilde{f}$ to f

2. Minimize the squared error between the observed response g and the response $\tilde{g}$ obtained with the input $\tilde{f}$

3. Find $\tilde{f}$ that minimizes the squared error ε^2

Inverse filtering [2]

The error between g and $\tilde{g}$ is given by

$$\varepsilon = g - \tilde{g} = g - h \cdot \tilde{f}$$

The squared error is given as

$$\varepsilon^2 = \varepsilon^T \varepsilon = (g - h\tilde{f})^T (g - h\tilde{f})$$
$$= g^T g - \tilde{f}^T h^T g - g^T h\tilde{f} + \tilde{f}^T h^T h\tilde{f}$$

Now, let's find $\tilde{f}$ that minimizes ε^2

$$\frac{\partial \varepsilon^2}{\partial \tilde{f}} = -2h^T g + 2h^T h\tilde{f}$$

Inverse filtering [3]

Setting this expression to zero, we get

$$\tilde{f} = (h^T h)^{-1} h^T g$$

If h is square, non-singular (i.e., invertible) and circulant (or block circulant), than the previous one becomes

$$\tilde{f} = h^{-1} g = W D_h^{-1} W^{-1} g, \text{ where :}$$

Inverse DFT

Point by point transform-domain filtering with the DFT of g

DFT of g (with a scale factor of 1/N)

W = matrix of the eigenvectors of h

D_h = diagonal matrix whose elements are the eigenvalues of h

Inverse filtering [4]

These considerations lead us to the formulation of the inverse filter,

$$\tilde{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

that may be expressed as

$$L_I(u,v) = \frac{1}{H(u,v)}$$

IMP: if $H(u,v)$ has zeros, the filter fails!

Moreover, if we have noise, we get

$$\tilde{F}(u,v) = F(u,v) + \frac{\eta(u,v)}{H(u,v)}$$

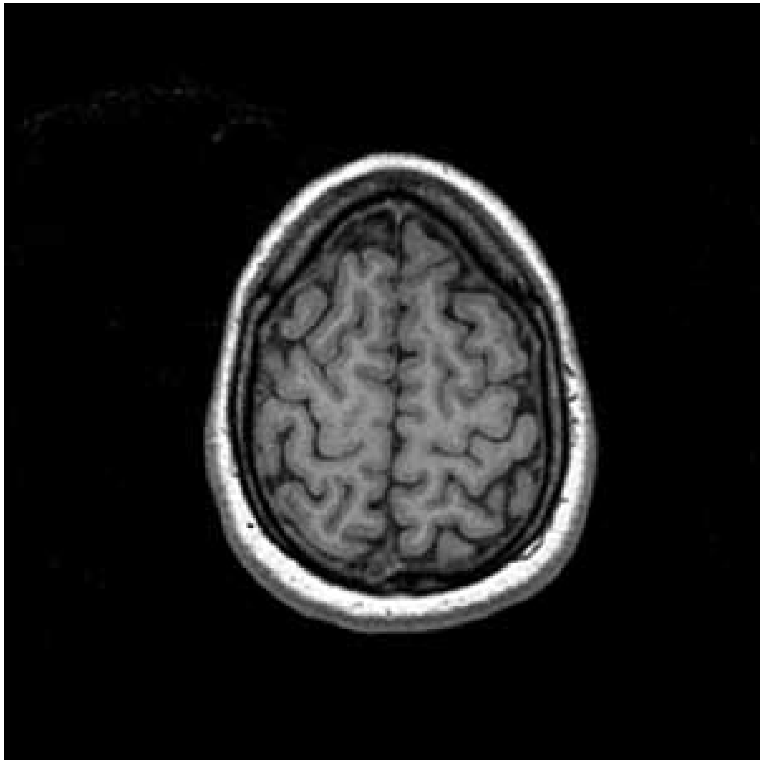
uniformly distributed

lowpass function

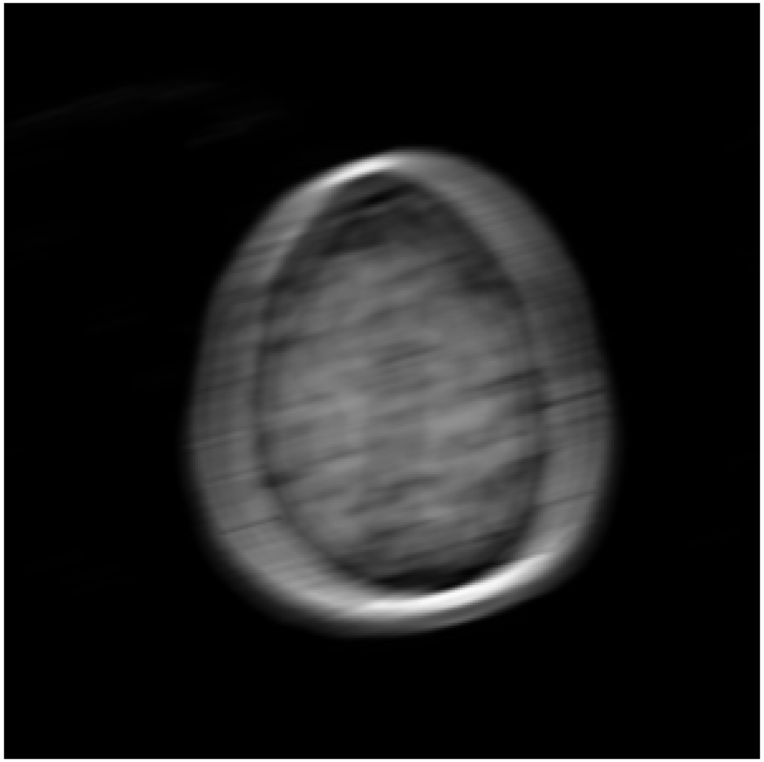
Image Restoration

Inverse filtering [5] - Example

original image



blurred image (length = 25, theta = 15)

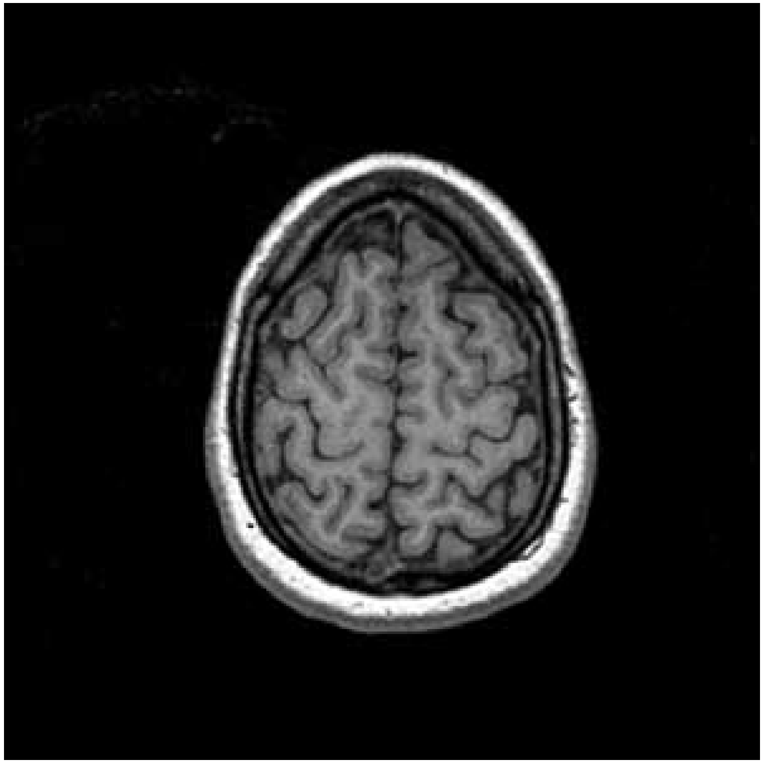


MSE = 28.1355

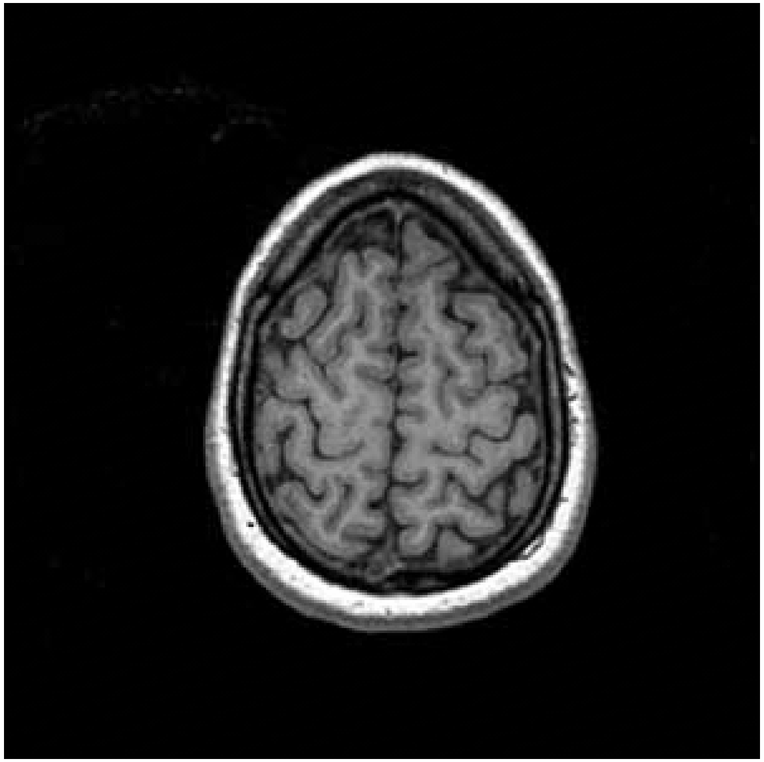
Image Restoration

Inverse filtering [6] - Example

original image



restored image (Inverse filtering)

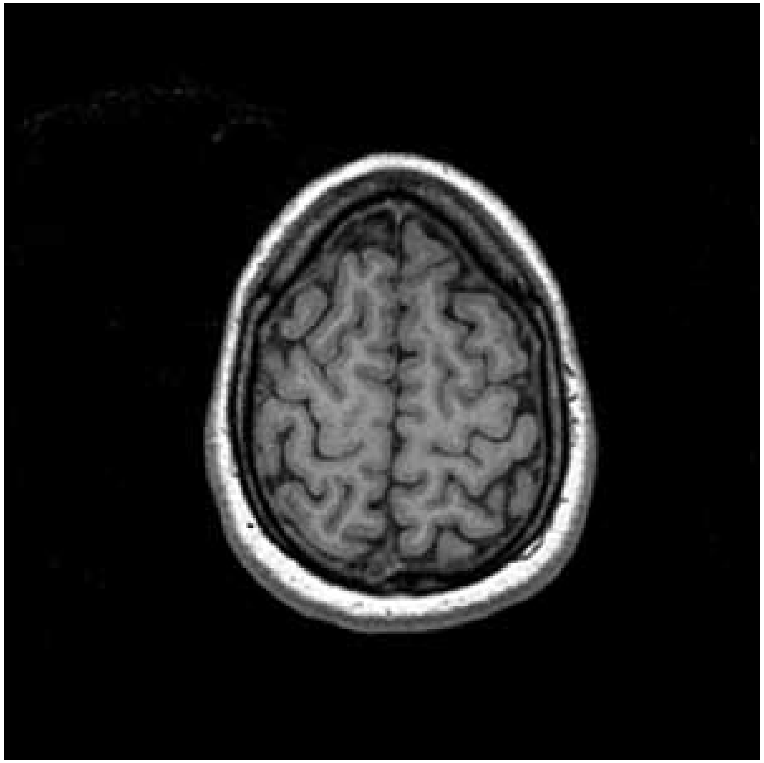


MSE = 0.0401

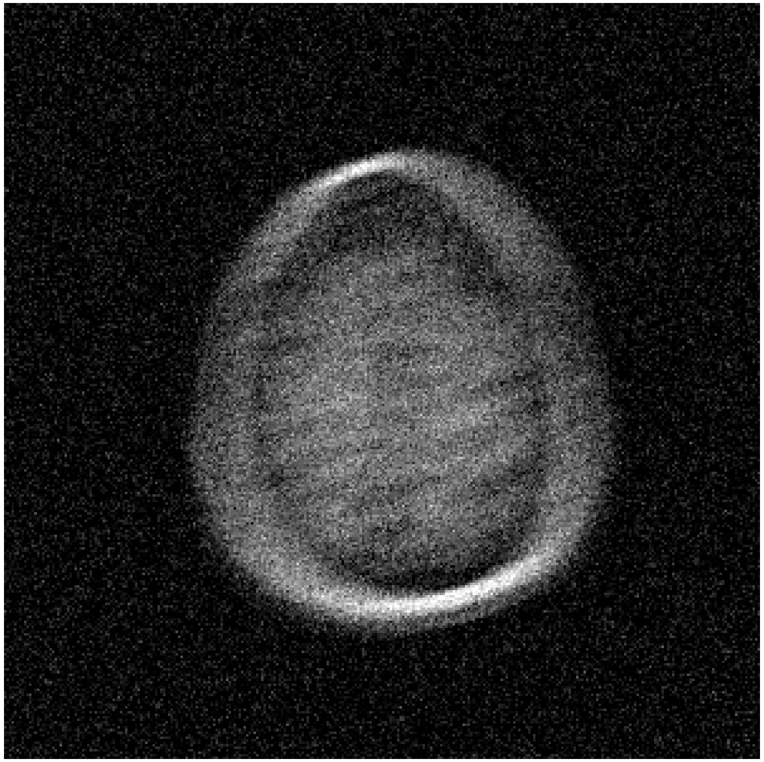
Image Restoration

Inverse filtering [7] - Example

original image



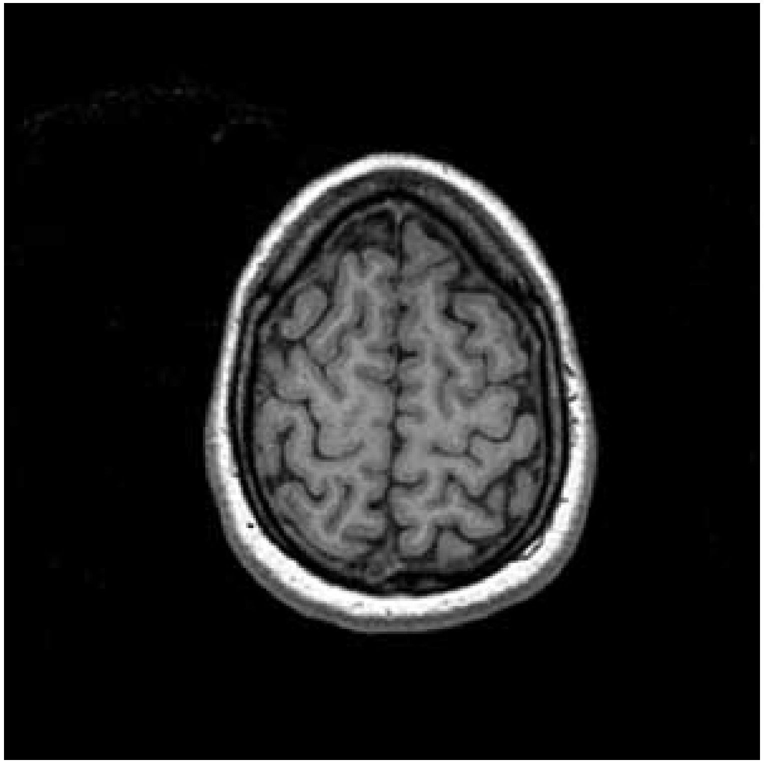
blurred image (length = 25, theta = 15)
+ gaussian noise (σ^2 = 0,01)



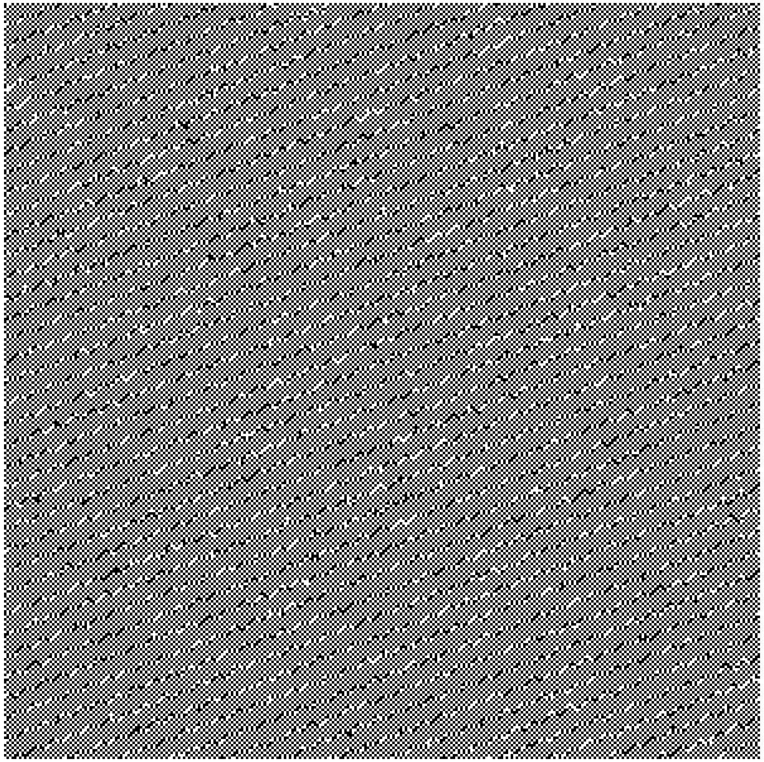
MSE = 29.3082

Inverse filtering [8] - Example

original image



restored image (Inverse filtering)



MSE = 32.4683

Power Spectrum Equalization (PSE)

Let's come back again to our degradation model:

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v) \cdot F(u,v) + \eta(u,v)$$

The model of power spectrum equalization (PSE) try to find a linear transform $\mathcal{L}$ in order to obtain an estimate

$$\tilde{f}(x,y) = \mathcal{L}[g(x,y)]$$

subject to the constraint

$$\Phi_{\tilde{f}}(u,v) = \Phi_{f}(u,v)$$

Power Spectrum Equalization (PSE) [2]

Applying $\mathcal{L}$ to the degradation model, we'll obtain

$$\Phi_{\tilde{f}}(u,v) = \left|L(u,v)\right|^2 \left[\left|H(u,v)\right|^2 \Phi_f(u,v) + \Phi_\eta(u,v)\right] = \Phi_f(u,v)$$

where $L(u,v)$ is the Modulation Transfer Function (MTF) of the filter $\mathcal{L}$.

Deriving $L(u,v)$ from this expression, the final result is

$$L_{PSE}(u,v) = \left|L(u,v)\right| = \left[\frac{1}{\left|H(u,v)\right|^2 + \dfrac{\Phi_\eta(u,v)}{\Phi_f(u,v)}}\right]^{1/2}$$

Power Spectrum Equalization (PSE) [3]

Characteristics of the PSE filter

$$L_{PSE}(u,v)=\left|L(u,v)\right|=\left[\frac{1}{\left|H(u,v)\right|^{\cancel{2}}+\underbrace{\frac{\Phi_{\eta}(u,v)}{\Phi_{f}(u,v)}}_{=0}}\right]^{\cancel{\frac{1}{2}}} \rightarrow =\frac{1}{H(u,v)}$$

- Requires knowledge of the PSDs of the original image and noise processes
- If the noise PSD tends to zero, the PSE filter tends to the inverse filter
- Only restoration in the spectral magnitude (no phase correction)
- Its gain is not affected by zeros in $H(u,v)$ as long as $\Phi_{\eta}(u,v)$ is also not zero at the same frequencies
- The gain of the PSE filter tends to zero wherever the original image PSD is zero (possibility to control the noise)

Image Restoration

The Wiener filter

Provides optimal filtering by taking into account the statistical characteristics of the image and noise processes

optimal = best achievable result under the conditions imposed and the information provided

The basic degradation model used is

$$g = h \cdot f + \eta$$

where f and η are stationary linear stochastic processes with known spectral characteristics or known autocorrelation

The Wiener filter [2]

Approach

We want to derive a filter L in order to obtain a linear estimate $\tilde{f} = Lg$ to f from the given image g

The criterion used to minimize the MSE is

$$\varepsilon^2 = \left[E \left\| f - \tilde{f} \right\|^2 \right]$$

First, we express the MSE as the trace of the outer product matrix of the error vector

$$\varepsilon^2 = E\left\{ Tr\left[\left(f - \tilde{f}\right)\left(f - \tilde{f}\right)^T \right] \right\}$$

The Wiener filter [3]

Approach

Because the trace of a sum of matrices is equal to the sum of their traces, the E and Tr operator may be interchanged.

Now, exploiting some relationships, we'll get that the MSE can be even written as

$$\varepsilon^2 = Tr(\phi_f - 2\phi_f h^T L^T + Lh\phi_f h^T L^T + L\phi_\eta L^T)$$

Note that here the MSE is no longer a function of f, g or η, but depends only on the statistical characteristics of f and η, as well as on h and L

The Wiener filter [4]

In order to derive the optimal filter L, we can calculate

$$\frac{\partial \varepsilon^2}{\partial L} = -2\phi_f h^T + 2Lh\phi_f h^T + 2L\phi_\eta = 0$$

which leads to the optimal Wiener filter function

$$L_W = \phi_f h^T (h\phi_f h^T + \phi_\eta)^{-1}$$

Problem: making the inversion of this matrix is not easy

Possible solution: let's try to write the matrix as a product of diagonal and unitary matrices

The Wiener filter [5]

First of all, we know that h is block circulant and that Φ_f can be usually approximated by a block circulant matrix

As a consequence,

$$h = WD_h W^{-1}$$

$$\Phi_f = WD_f W^{-1}$$

Moreover, Φ_η is a diagonal matrix if η is white:

$$\Phi_\eta = WD_\eta W^{-1}$$

The Wiener filter is then given by

$$L_W = WD_f D_h^* (D_h D_f D_h^* + D_\eta)^{-1} W^{-1}$$

Image Restoration

The Wiener filter [6]

So, the Minimum Mean Squared Error (MMSE) estimate is given by

$$\tilde{f} = L_W g = W D_f D_h^* (D_h D_f D_h^* + D_\eta)^{-1} W^{-1} g$$

Transfer function $H(u,v)$ of the degrading system

PSD $\Phi_f(u,v)$ of the original image

PSD $\Phi_\eta(u,v)$ of the noise process

DFT of g (with a scale factor of 1/N)

$$\tilde{F}(u,v) = \frac{\Phi_f(u,v) H^*(u,v) G(u,v)}{H(u,v)\Phi_f(u,v)H^*(u,v) + \Phi_\eta(u,v)} = \left[\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + \dfrac{\Phi_\eta(u,v)}{\Phi_f(u,v)}} \right] G(u,v)$$

Image Restoration

The Wiener filter [7]

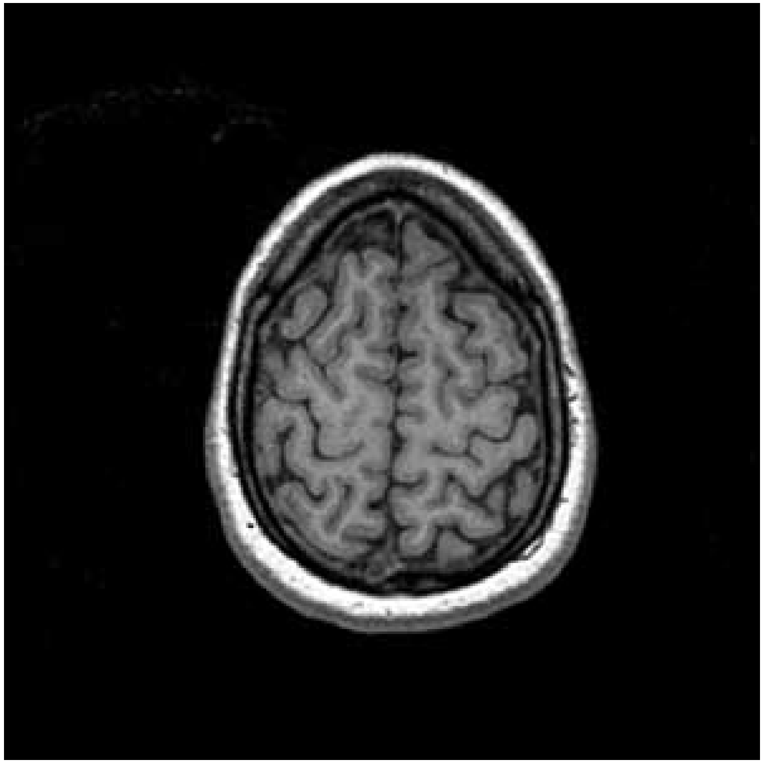
$$L_W(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \frac{\Phi_\eta(u,v)}{\Phi_f(u,v)}} \right]$$

- Without noise, we have $\Phi_\eta(u,v)=0$, and this filter reduces to the inverse filter
- This is the inverse of the SNR ratio; consequently, also if the SNR is high the Wiener filter is close to the inverse
- If there is no blurring (i.e. $H(u,v)=1$), then

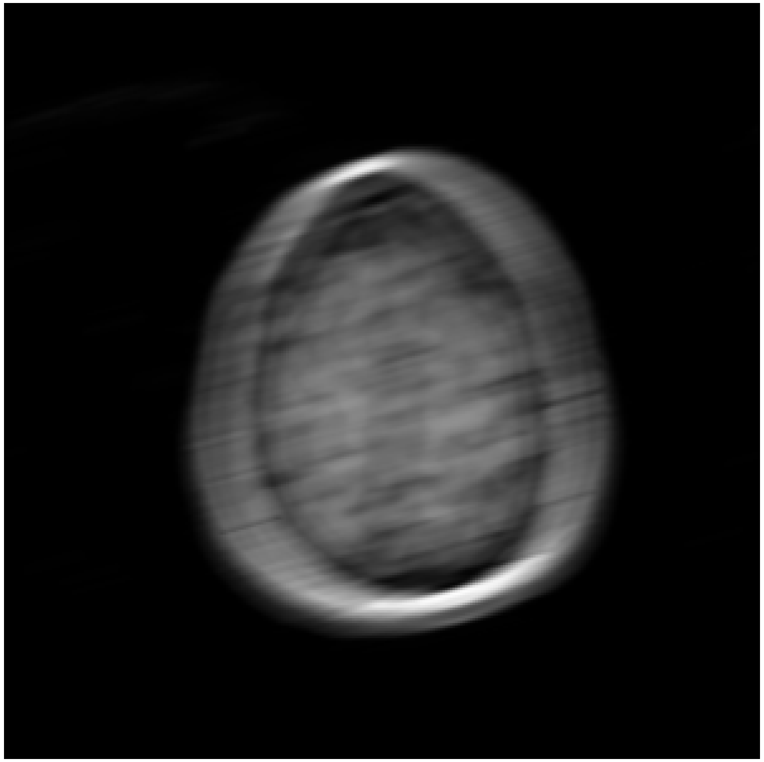
$$L_W(u,v) = \left[\frac{\Phi_f(u,v)}{\Phi_f(u,v) + \Phi_\eta(u,v)} \right]$$

The Wiener filter [8] - Example

original image



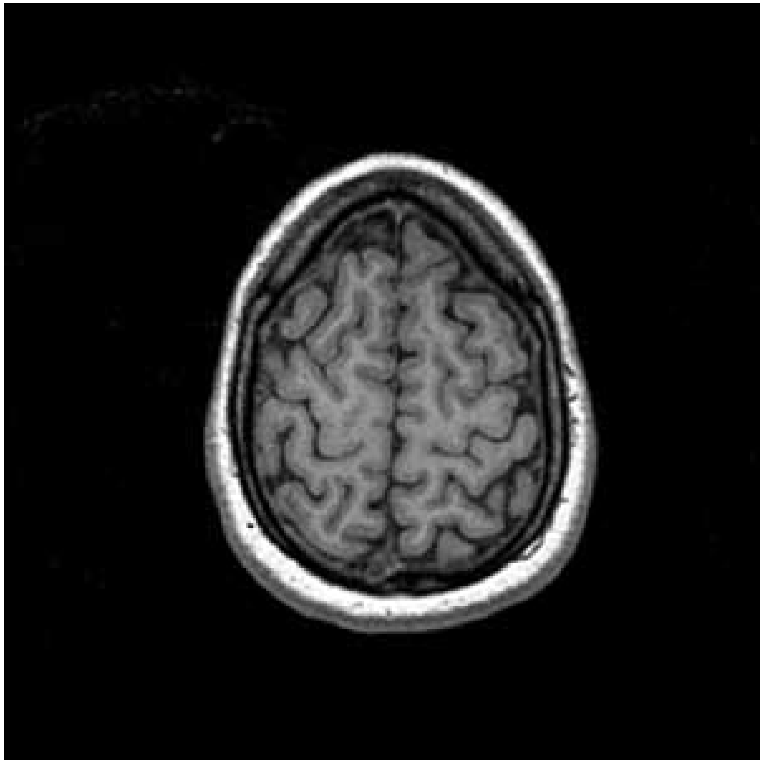
blurred image (length = 25, theta = 15)



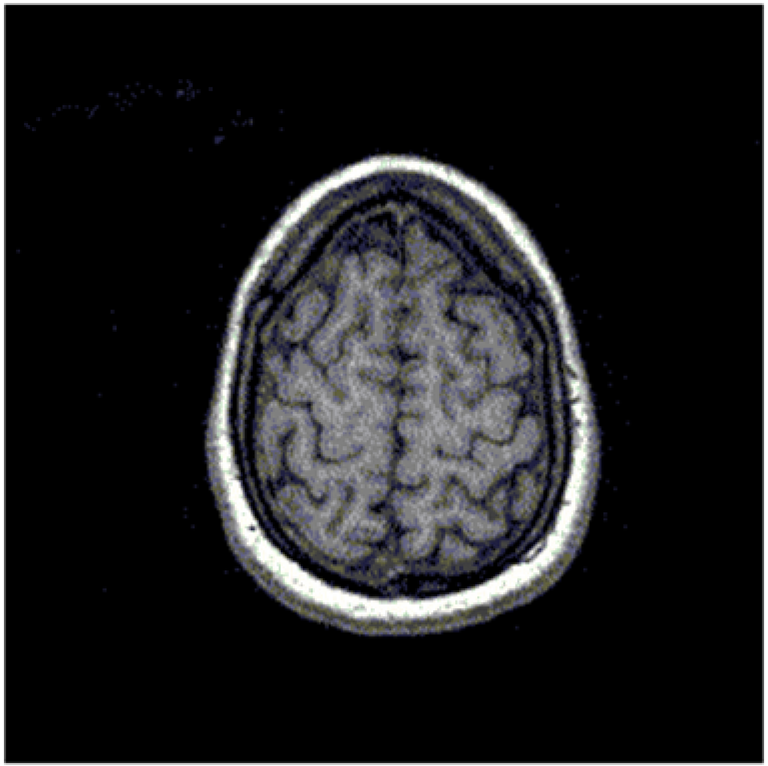
MSE = 28.1355

The Wiener filter [9] - Example

original image



restored image (Wiener filtering)

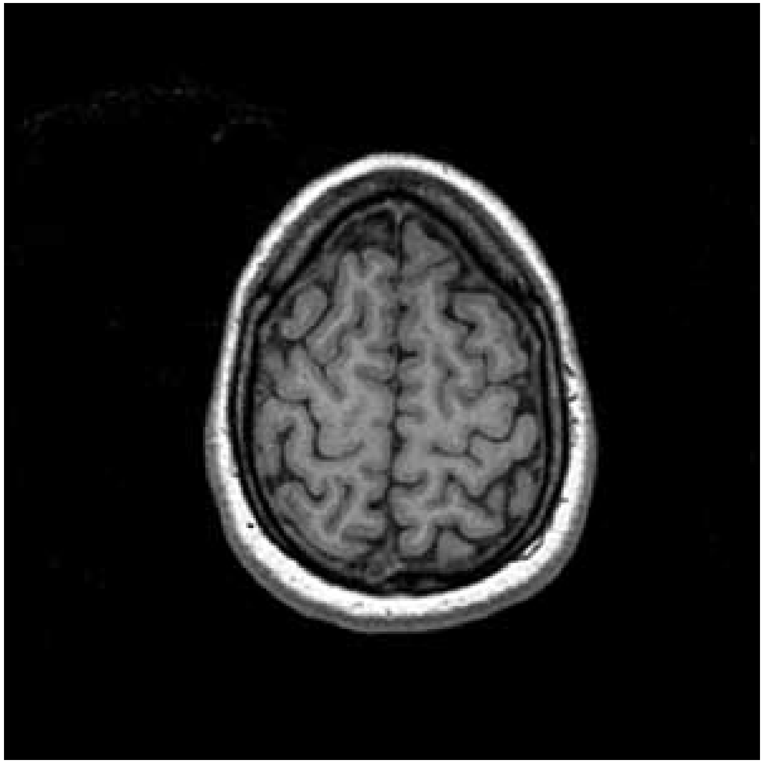


MSE = 12.3747

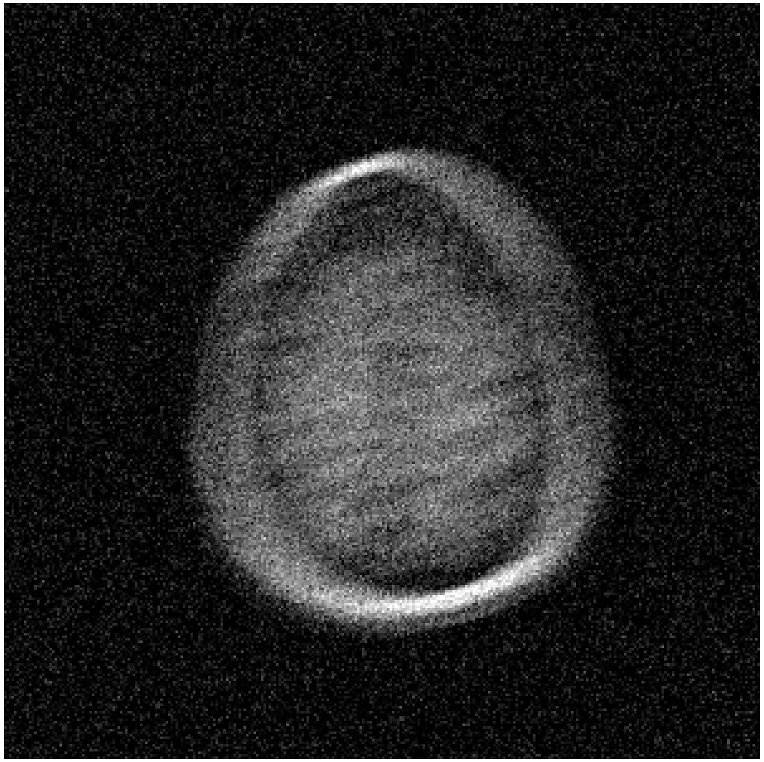
Image Restoration

The Wiener filter [10] - Example

original image



blurred image (length = 25, theta = 15) + gaussian noise (σ^2 = 0,01)

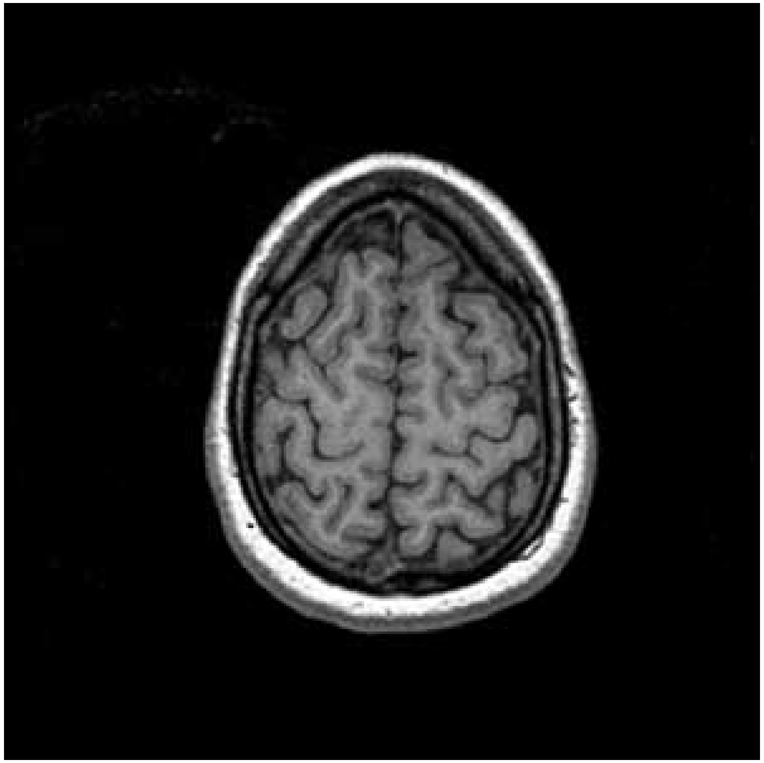


MSE = 29.3082

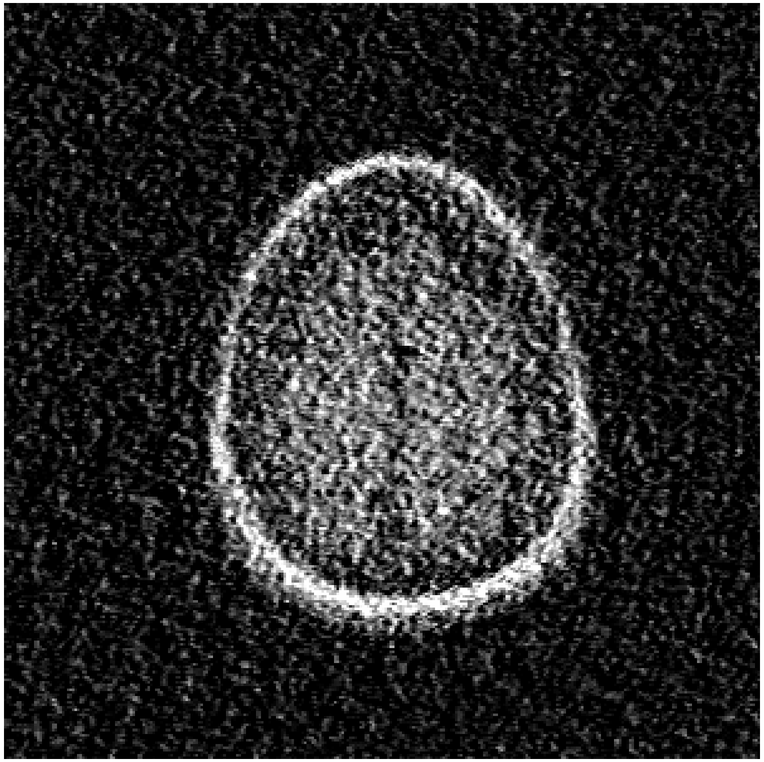
Image Restoration

The Wiener filter [11] - Example

original image



restored image (Wiener filtering)



MSE = 29.5374

Comparative Analysis: inverse, PSE, Wiener

- When the noise PSD is zero, the PSE filter is equivalent to the inverse filter
- When the noise PSD is zero, the Wiener filter is equivalent to the inverse filter
- The gains of the inverse, PSE and Wiener filters are related as $|L_I(u,v)| > |L_{PSE}(u,v)| > |L_W(u,v)|$
- The PSE filter is the geometric mean of the inverse and Wiener filters, $L_{PSE}(u,v) = [L_I(u,v)L_W(u,v)]^{1/2}$
- Because $|L_{PSE}(u,v)| > |L_W(u,v)|$, the PSE filter admits more high-frequency components with larger gain than the Wiener
- The PSE filter doesn't have a phase component. Phase correction, if required, must be applied separately

Constrained least-squares restoration

Wiener filter is optimal, generally, only for the class of images represented by the statistical entities used, but it can be unsatisfactory for other specific images

The Constrained least-squared restoration (CLSR), instead, is an optimal procedure for every specific image given, under particular constraints that are imposed

Called L a linear filter operator and using again the degradation model $g = h\,f + \eta$, the restoration problem is to minimize $\left\| L\tilde{f} \right\|^2$ subject to $\left\| g - h\tilde{f} \right\|^2 = \left\| \eta \right\|^2$

Image Restoration

Constrained least-squares restoration [2]

Using the method of Lagrange multipliers, we want to find $\tilde{f}$ that minimizes the function

$$J(\tilde{f}) = \left\| L\tilde{f} \right\|^2 + \alpha \left[\left\| g - h\tilde{f} \right\|^2 - \left\| \eta \right\|^2 \right]$$

where α is the Lagrange multiplier.

Taking the derivative of $J(\tilde{f})$ with respect to $\tilde{f}$ and setting it equal to zero, at the end we get

$$\tilde{f} = (h^T h + \gamma L^T L)^{-1} h^T g$$

where $\gamma = \dfrac{1}{\alpha}$

Constrained least-squares restoration [3]

Using the Laplacian operator, we can construct L as a block circulant matrix

Now, L is diagonalized by the 2D DFT as $D_L = W^{-1}LW$, where D_L is a diagonal matrix

Exploiting this property, at the end we get

$$\tilde{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |L(u,v)|^2} \right] G(u,v)$$

where $L(u,v)$ is the transfer function related to the constraint operator L

Image Restoration

Constrained least-squares restoration [4]

$$\tilde{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |L(u,v)|^2} \right] G(u,v)$$

Considerations:

- The PSDs of the image and noise processes are not required
- It's necessary to have an estimate of the mean and of the variance of the noise process (in order to determine the optimal value for γ)
- If $\gamma = 0$, the filter reduces to the inverse filter

Constrained least-squares restoration [5]

How to determine γ ?

Let's define a residual vector as $r = g - h\tilde{f}$

We want to find γ such that $\|r\|^2 = \|\eta\|^2 \pm \varepsilon$

factor of accuracy

total energy of the noise process

1. Choose an initial value for γ
2. Compute $\tilde{F}(u,v)$ and $\tilde{f}$
3. Form the residual vector r and compute $\|r\|^2$
4. Increment γ if $\|r\|^2 < \|\eta\|^2 - \varepsilon$, decrement it if $\|r\|^2 > \|\eta\|^2 + \varepsilon$ and return to step 2. Stop if $\|r\|^2 = \|\eta\|^2 \pm \varepsilon$

Image Restoration

The Metz filter

Modification of the inverse filter for application to nuclear medicine images, including noise suppression at high frequencies

lowpass filter highpass filter lowpass filter

$$L_M(u,v) = \frac{1-\left[1-H^2(u,v)\right]^{\chi}}{H(u,v)}$$

- χ is a factor that controls how much up with the frequencies we can go to have a predominance of the inverse filter (after that, the noise-suppression feature becomes stronger)
- χ can be selected in order to minimize the MSE between the filtered and the ideal images

Image Restoration

Information required for image restoration

INVERSE FILTER

$$L_I(u,v) = \frac{1}{H(u,v)}$$

- MTF of the degradation process
- PSF

POWER SPECTRUM EQUALIZATION

$$L_{PSE}(u,v) = \left[\frac{1}{\left|H(u,v)\right|^2 + \dfrac{\Phi_\eta(u,v)}{\Phi_f(u,v)}} \right]^{\frac{1}{2}}$$

- MTF
- PSD of the noise
- PSD of the original image

WIENER FILTER

$$L_W(u,v) = \left[\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + \dfrac{\Phi_\eta(u,v)}{\Phi_f(u,v)}} \right]$$

- PSF
- Original image and noise autocorrelation matrices

Deblurring

Blind Deblurring

- Sometimes it's not possible to obtain distinct models of the degradation phenomena
- We need to derive information from the degraded image

$$\Phi_g(u,v) = |H(u,v)|^2 \Phi_f(u,v) + \Phi_\eta(u,v)$$

- PSE: divide the given degraded image N x N into M x M segments $g_l(m,n), \quad l = 1,2,\ldots,Q^2$ where $Q = N/M$

$$\Phi_{g_l}(u,v) = |H(u,v)|^2 \Phi_{f_l}(u,v) + \Phi_{\eta_l}(u,v)$$

  - Blurring across the boundaries of adjacent subimages is ignored
  - Average PSDs Φ_{g_l} over all the Q^2 available segments

$$\frac{1}{Q^2}\sum_{l=1}^{Q^2}\Phi_{g_l}(u,v) = |H(u,v)|^2 \tilde{\Phi}_f(u,v) + \tilde{\Phi}_\eta(u,v)$$

DENOMINATOR OF L_{PSE}

  - No necessity to know MTF & PSD of the noise, but only Φ_f

Deblurring

Iterative Blind Deblurring

- Assumptions:
  - MTF of the LSI system causing the degradation has zero phase
  - The magnitude of the LSI system is a smoothly varying function of frequency
- The Fourier representation of a signal is affected by the blur function but edge locations don't change in the phase
- <u>Method of Rabie</u>: try to recover the original magnitude spectrum using the edge information preserved in the phase

Deblurring

Method of Rabie

$$M_g(u,v) = M_f(u,v) M_h(u,v) \qquad M_f(u,v), M_g(u,v) \text{ spectral magnitudes}$$

$$\theta_g(u,v) = \theta_f(u,v) \qquad M_h(u,v) \text{ degradation MTF (zero phase)}$$

1. $M_g(u,v)$ is smoothed with the assumption that $M_h(u,v)$ is smooth
2. $M_f(u,v)$ is initially approximated like

$$M_f(u,v) \cong M_g(u,v) \frac{S[M_f(u,v)]}{S[M_g(u,v)]}, \qquad S \text{ smoothing operator}$$

3. Iterate the formula to refine $M_f(u,v)$: $\tilde{M}_f^{l+1} = M_g \dfrac{S[\tilde{M}_f^l]}{S[M_g]}$

Note: this method uses the entire image, not sections!

Deblurring

Method of Rabie [2]

<u>Problem</u>: in this way a unit constant is being added to the spectral magnitude at all frequencies: infact

$$\tilde{M}_f^0 = M_g + 1$$

- Production of a noisy initial approximation of M_f (amplification of the high frequency components)
- M_f can be better approximated by the formula

$$\tilde{M}_f^0 = M_g + \frac{M_f}{S[M_f]}$$

Deblurring

Method of Rabie [3] - Summary

- Obtain an initial estimate of M_f with

$$\tilde{M}_f^0 = M_g + \frac{M_f}{S[M_f]}$$

- Update the estimate iteratively using $\tilde{M}_f^{l+1} = M_g \frac{S[\tilde{M}_f^l]}{S[M_g]}$

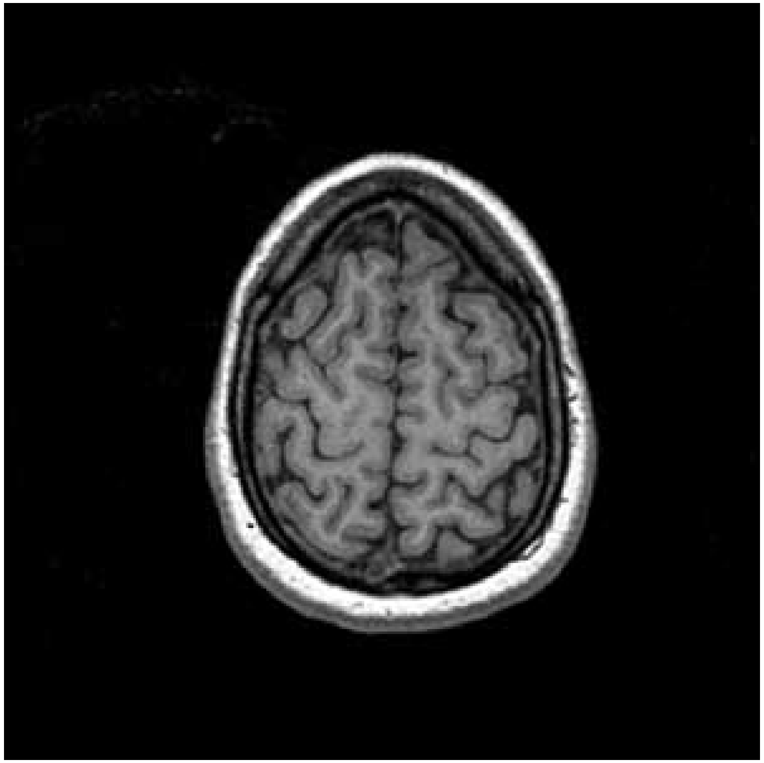
- Stop when the MSE between two consecutives iterations is less than a certain limit
- Combine the best estimate of M_f with the phase function, obtaining the Fourier transform of the restored image

$$\tilde{F}(u,v) = \tilde{M}_f(u,v)\exp[j\theta_f(u,v)]$$

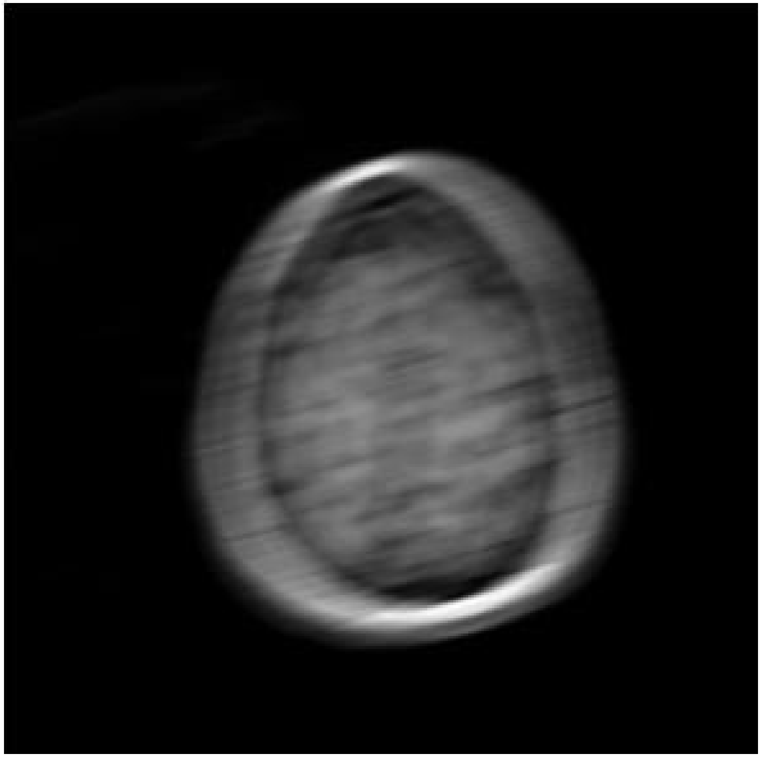
Image Restoration

Iterative Blind Deblurring - Example

original image



blurred image (length = 25, theta = 15)

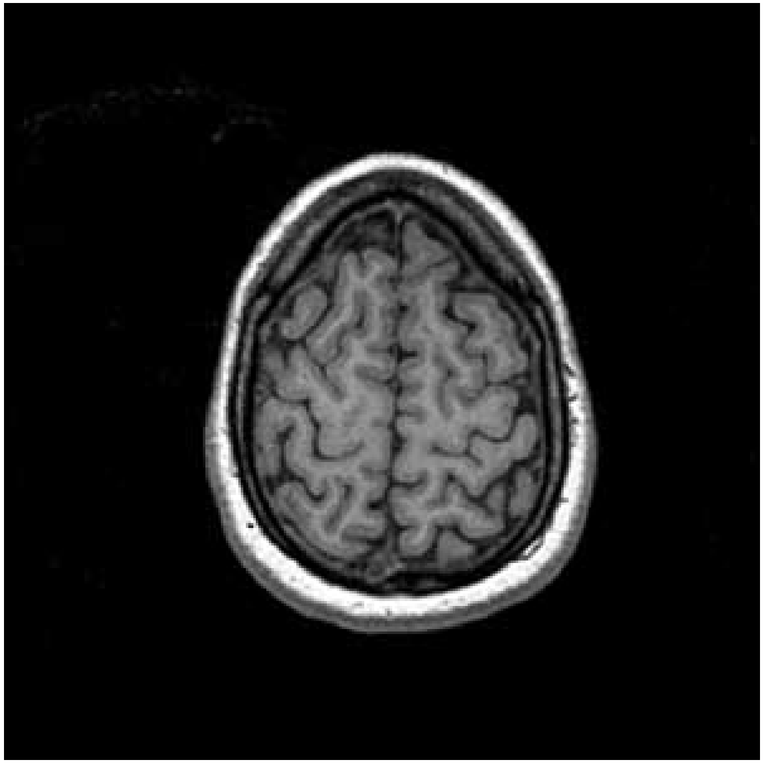


MSE = 28.1355

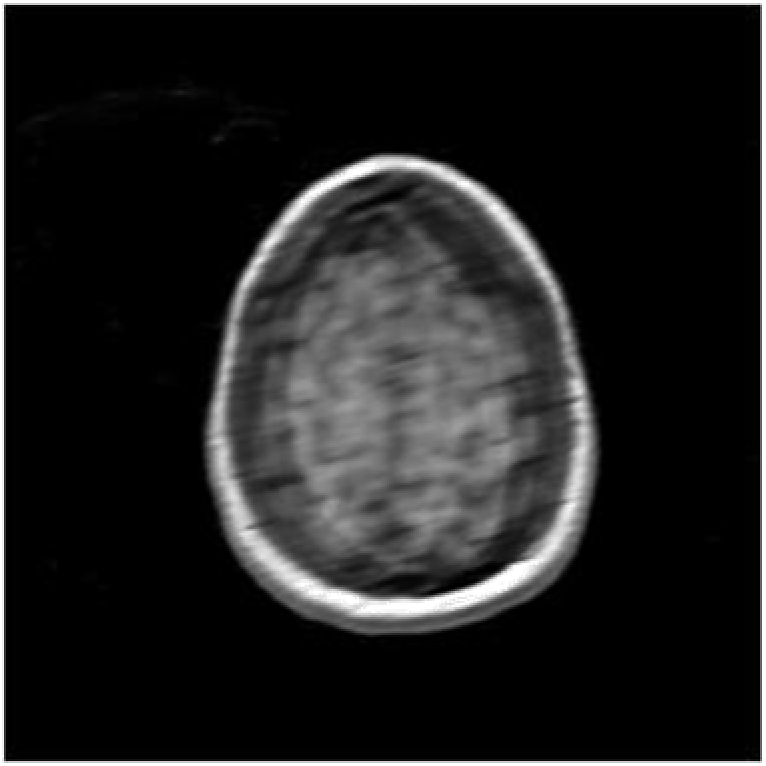
Image Restoration

Iterative Blind Deblurring - Example

original image



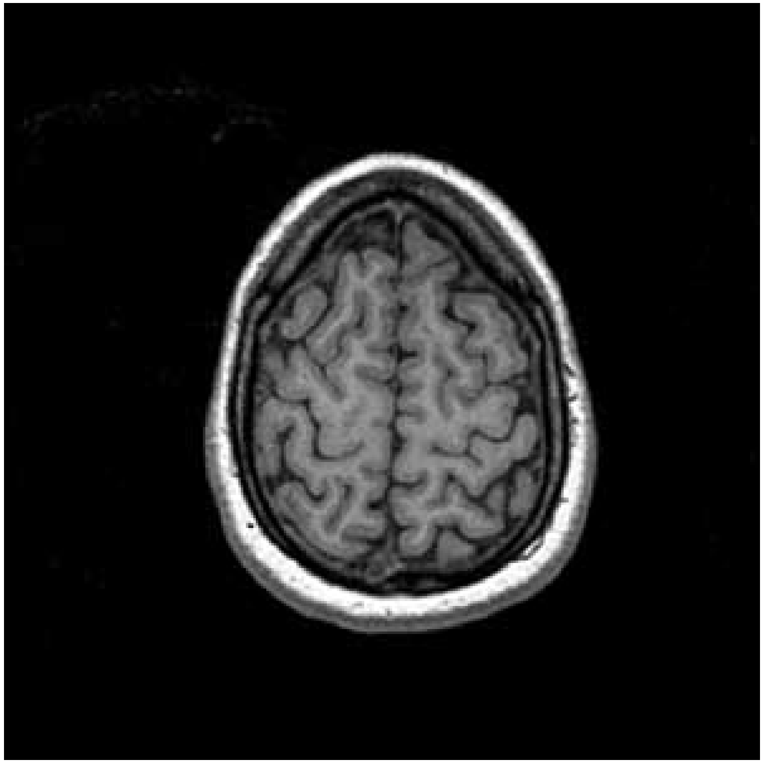
Restored Image, NUMIT = 10



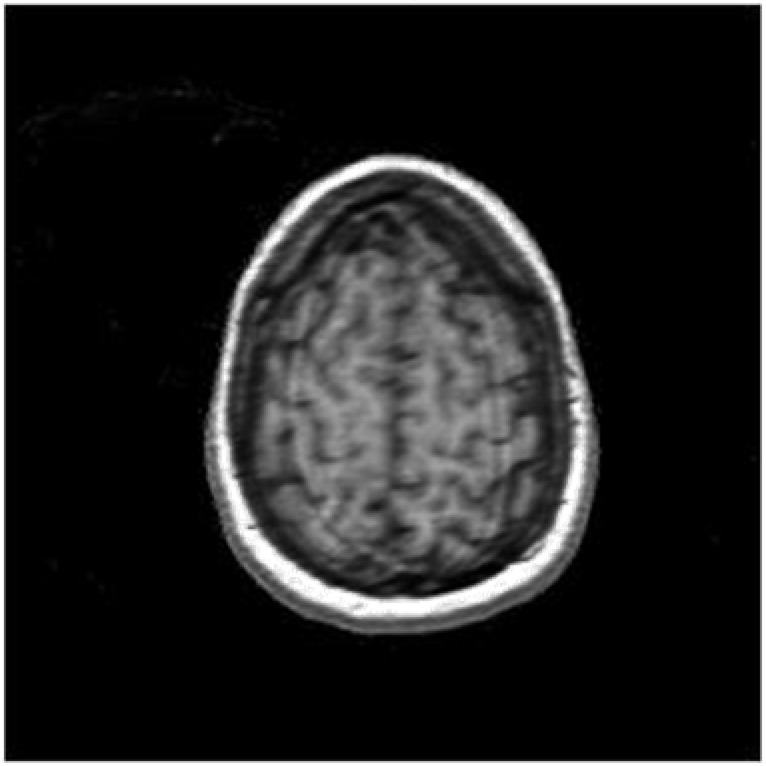
MSE = 21.6977

Iterative Blind Deblurring - Example

original image



Restored Image, NUMIT = 25

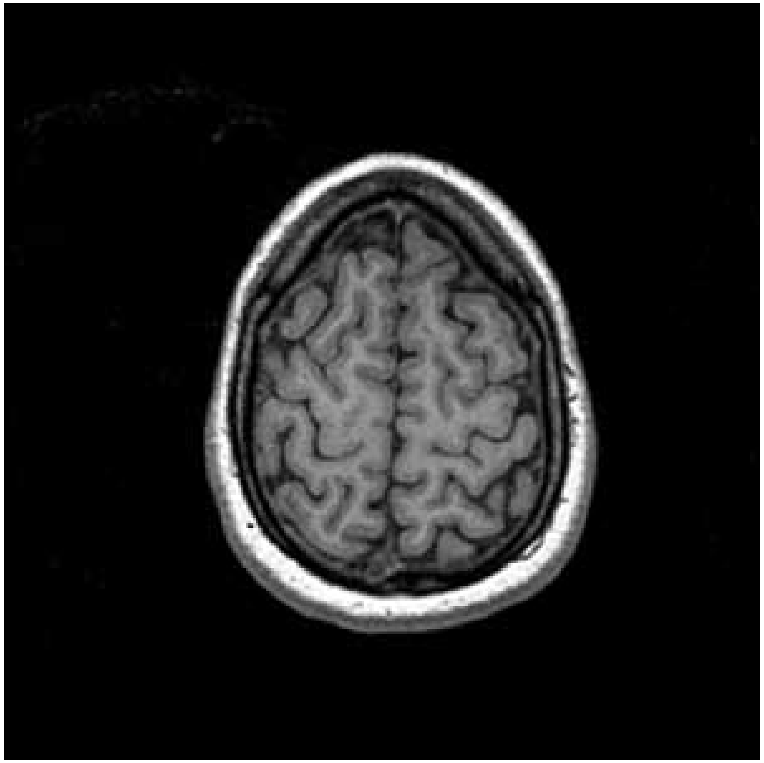


MSE = 15.5829

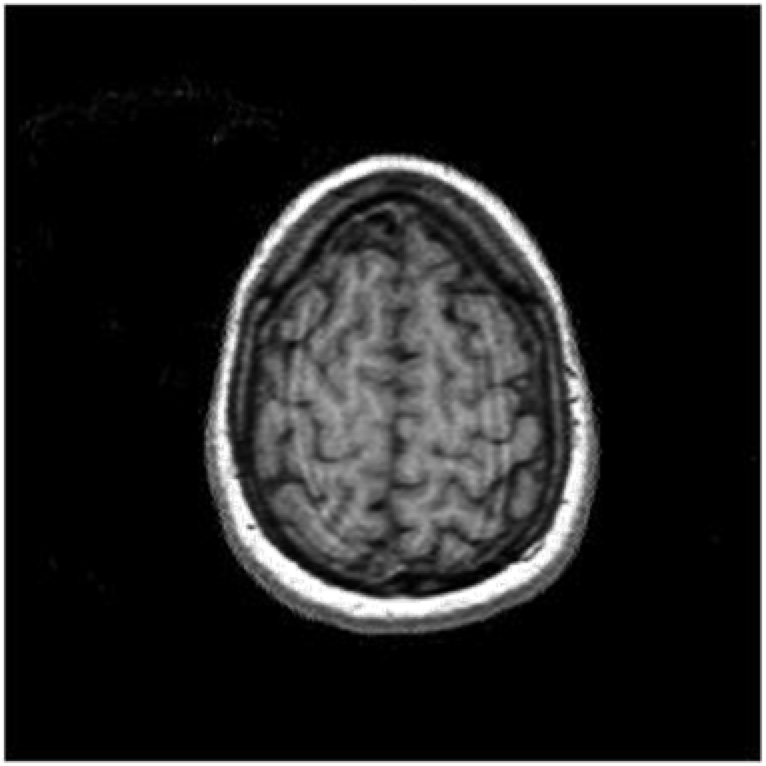
Image Restoration

Iterative Blind Deblurring - Example

original image



Restored Image, NUMIT = 50



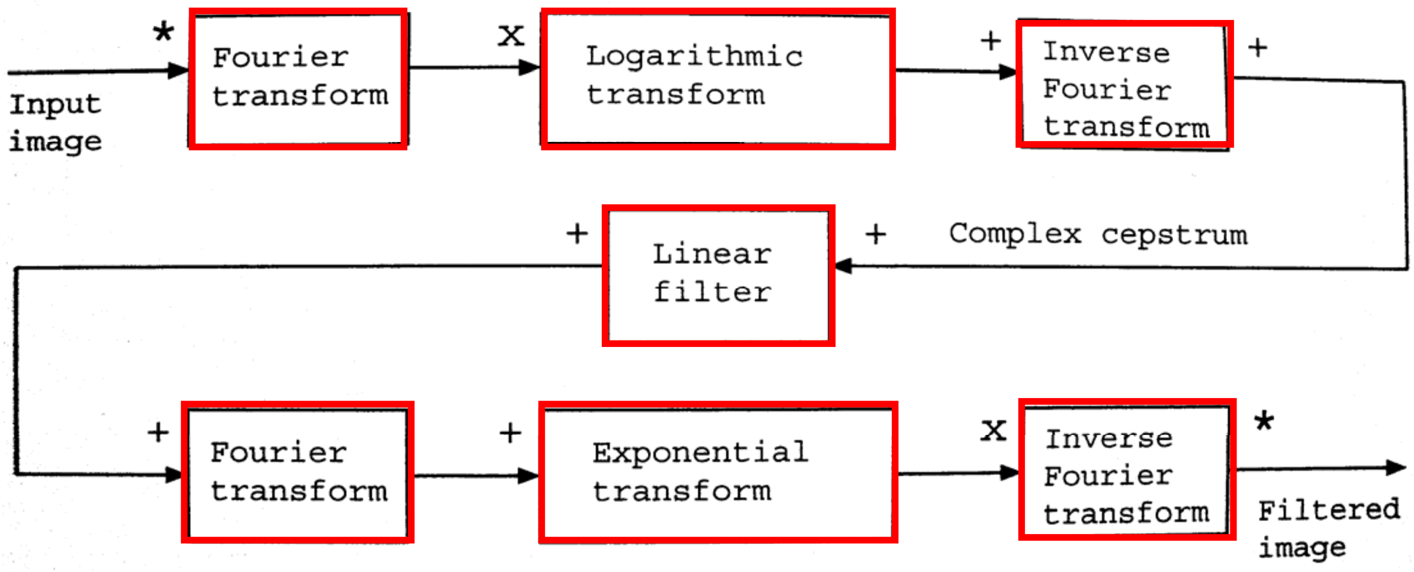
MSE = 12.2869

Deconvolution

Homomorphic Deconvolution

An image that is given by the convolution of two images:

$$g(x,y) = h(x,y) * f(x,y)$$



Note: Homomorphic convolution can be used, for example, to remove repeated versions of a basic pattern in an image (visual echo)

Image Restoration

Space-variant Restoration

- Problems of restoration techniques like PSE, Wiener:
  - Assumption that the image can be modeled by a stationary (random) field;
  - Necessity to know the PSD of the image

  - The deblurred image suffer from artifacts at the boundaries

Several adaptive techniques for space-variant restoration have been proposed:

- Sectioned image restoration
- Adaptive-neighborhood deblurring
- Kalman filter

Image Restoration

Sectioned image restoration

- The input image is divided into small *P x P* sections
- For each section, the MAP (maximum-a-posteriori probability) is estimated → suppression of the noise
- Each small section is now close to a stationary process → Wiener, PSE
- Combine all the sections together to form the final deblurred image
- Limitations:
  - Stationarity of each section may not be satisfied
  - Sections cannot be arbitrarily small (must be larger than the Region of Support of the blur PSF) → artifacts could arise at the section boundaries

Adaptive-neighborhood deblurring

- The input image is treated as being made up of a collection of regions of relatively uniform gray levels
- An adaptive neighborhood is determined for each pixel in the image
- Assuming that each region is larger than the Region of Support of the PSF,

$$g_{m,n}(p,q) \simeq h(p,q) * f_{m,n}(p,q) + \eta_{m,n}(p,q)$$

- Next, each adaptive-neighborhood region is centred within a rectangular region of the same size as the input image, and the area surrounding the region is padded with its mean in order to reduce edge artifacts

Adaptive-neighborhood deblurring [2]

- FFT

$$G_{m,n}(u,v) \simeq H(u,v)F_{m,n}(u,v) + \eta_{m,n}(u,v)$$

- 2D Hamming window $w_H(p,q)$

$$w_H(p,q) = \left[0.54 - 0.46\cos\left(\frac{2\pi \cdot p}{M-1}\right)\right]\left[0.54 - 0.46\cos\left(\frac{2\pi \cdot q}{M-1}\right)\right]$$

$$g_{m,n}(p,q)w_H(p,q) \simeq [h(p,q) * f_{m,n}(p,q)]w_H(p,q) + \eta_{m,n}(p,q)w_H(p,q)$$

- Noise estimation

$$\tilde{\eta}_{m,n}(u,v) = A_{m,n}(u,v)G_{m,n}(u,v)$$

where $A_{m,n}(u,v)$ is a frequency domain, magnitude-only scale factor that depends on the spectral characteristics of the adaptive-neighborhood region grown

Image Restoration

Adaptive-neighborhood deblurring [3]

$$\tilde{F}_{m,n}(u,v) = \frac{G_{m,n}(u,v)}{H(u,v)}[1 - A_{m,n}(u,v)]$$

- Imposing the PSDs of the original and of the estimate noise to be equal, we get

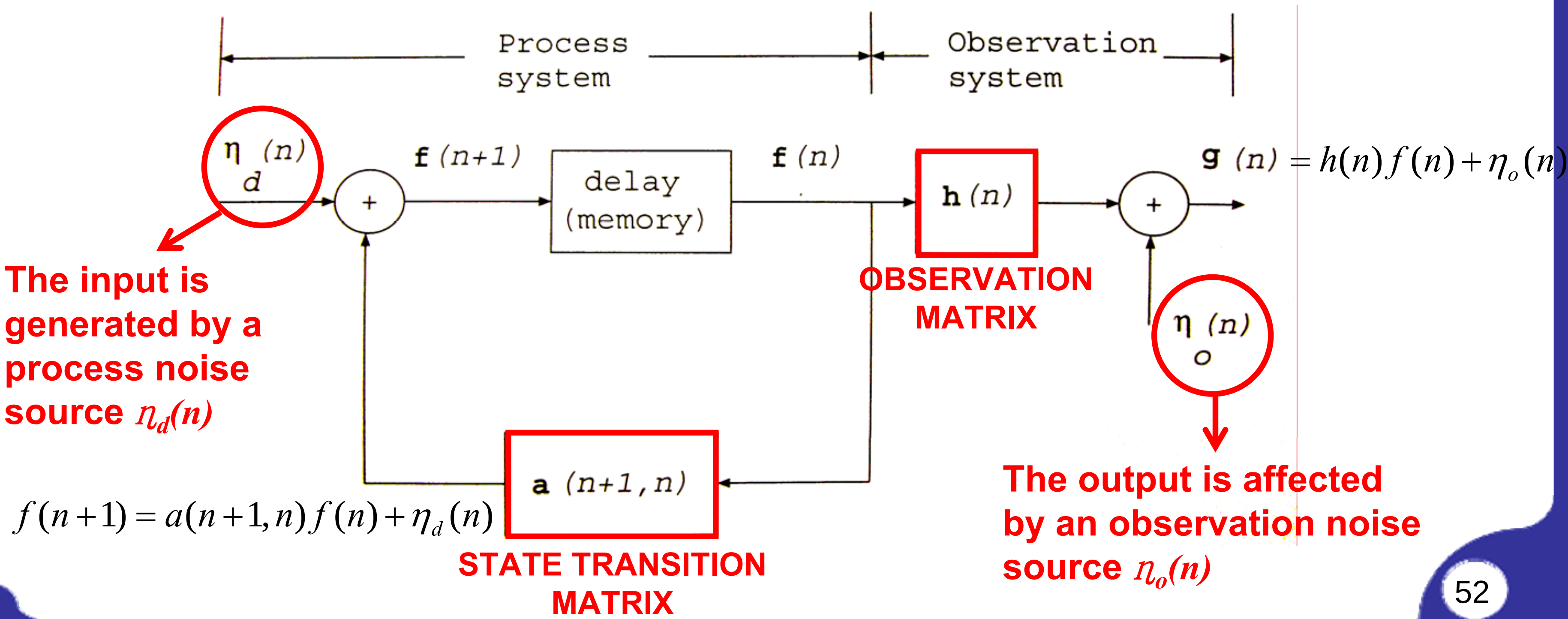
$$\Phi_{\eta_{m,n}}(u,v) = A^2_{m,n}(u,v)\Phi_{g_{m,n}}(u,v)$$

$$A_{m,n}(u,v) = \left(\frac{\Phi_\eta(u,v)}{|H(u,v)|^2 \Phi_{f_{m,n}}(u,v) + \Phi_\eta(u,v)} \right)^{1/2}$$

$$\boxed{\tilde{F}_{m,n}(u,v) = \frac{G_{m,n}(u,v)}{H(u,v)} \left[1 - \left(\frac{\Phi_\eta(u,v)}{\Phi_{g_{m,n}}(u,v)} \right)^{1/2} \right]}$$

Kalman filter

- The signals or items of information involved are represented as a state vector *f(n)* and an observation vector *g(n)*



Kalman filter [2]

$$g(n) = h(n) f(n) + \eta_o(n)$$

- Kalman filtering problem: given a series of the observations $G_n = \{g(1), g(2), \ldots, g(n)\}$ for each $n \geq 1$, find the MMSE estimate of the state vector $f(l)$
- The innovation process
  - Suppose that, after $n\text{-}1$ observations $g(1), g(2), g(n\text{-}1)$, the MMSE estimate $\tilde{f}(n-1|G_{n-1})$ of $f(n\text{-}1)$ has been obtained
  - Given a new observation $g(n)$, we could update the previous vector and obtain a new state vector $\tilde{f}(n|G_n)$
  - Bacause $f(n)$ and $g(n)$ are related via the observation system, defined $\tilde{g}(n|G_{n-1})$ the estimate of $g(n)$ given $G_{n\text{-}1}$, the innovation process is

$$\zeta(n) = g(n) - \tilde{g}(n|G_{n-1}), \quad n = 1,2,\ldots$$

Kalman filter [3] - Summary

- Data available: the observation vectors $G_n = \{g(1), g(2), ..., g(n)\}$
- System parameters assumed to be known:
  - The state transition matrix $a(n+1,n)$
  - The observation system matrix $h(n)$
  - The AutoCorrelation Function matrix of the driving noise $\phi_{\eta_d}(n)$
  - The ACF matrix of the observation noise $\phi_{\eta_o}(n)$
- Initial conditions:
  - $\tilde{f}(1|G_0) = E[f(1)] = 0$
  - The ACF matrix of the predicted state error, $\phi_{\varepsilon_p}(n+1,n)$, is diagonal per $n=0$

$$\phi_{\varepsilon_p}(1,0) = D_0$$

Kalman filter [4] – Computational Steps

1. Compute the Kalman gain matrix as

$$K(n) = a(n+1, n)\phi_{\varepsilon_p}(n, n-1)h^T(n)\left[h(n)\phi_{\varepsilon_p}(n, n-1)h^T(n) + \phi_{\eta_o}(n)\right]^{-1}$$

2. Obtain the innovation process vector using

$$\zeta(n) = g(n) - h(n)\tilde{f}(n|G_{n-1})$$

3. Update the estimate of the state vector as

$$\tilde{f}(n+1|G_n) = a(n+1, n)\tilde{f}(n|G_{n-1}) + K(n)\zeta(n)$$

4. Compute the ACF matrix of the filtered state error as

$$\phi_{\varepsilon_p}(n) = \phi_{\varepsilon_p}(n, n-1) - a(n, n+1)K(n)h(n)\phi_{\varepsilon_p}(n, n-1)$$

5. Update the ACF matrix of the predicted state error as

$$\phi_{\varepsilon_p}(n+1, n) = a(n+1, n)\phi_{\varepsilon_p}(n)a^T(n+1, n) + \phi_{\eta_d}(n)$$

Image Restoration

Restoration of Nuclear Medicine Images

Nuclear medicine images are useful in functional imaging of several organs, but are severely affected by several factors that degrade their quality and resolution.

Causes of degradation:

- Poor quality control → Blurring
- Poor statistics → Low SNR
- Photon-counting (Poisson) noise → Noise amplification
- Gamma-ray attenuation → Attenuating effect
- Compton scattering → Background noise
- Poor spatial resolution → Low efficiency in photon detection

Review: what is SPECT?

- Acronym for Single Photon Emission Computerized Tomography
- A radioactive isotope is bound to a substance that is readily taken up by the cells in the brain
- A small amount of this compound is injected into the patient's vein and is taken up by certain receptor sites in the brain. The patient then lies on a table for 14-16 minutes while a SPECT "gamma" camera rotates slowly around his head
- A supercomputer then reconstructs 3-D images of brain activity levels

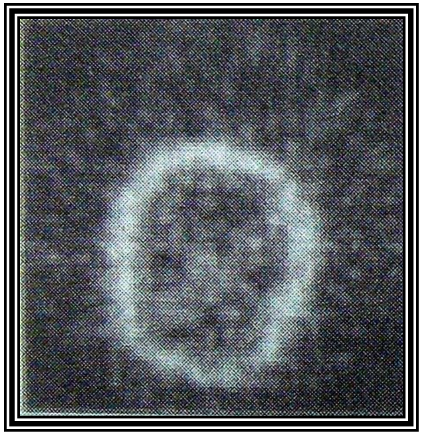
SPECT

Example: SPECT images of the brain

- Radioactive isotope: ^{99m}*Tc-chloride*
- 44 planar projections, each of size 64x64 pixels
  - Radius of rotation: 20cm
  - Time for acquisition: 30s

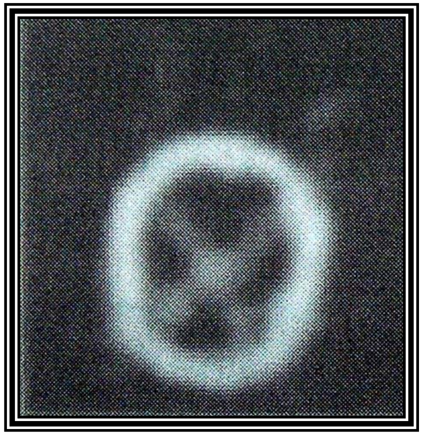
Transverse SPECT images have been reconstructed after performing geometric averaging of conjugate projections and restoration using the Wiener, PSE, and Metz filters

Example: SPECT images of the brain [2]

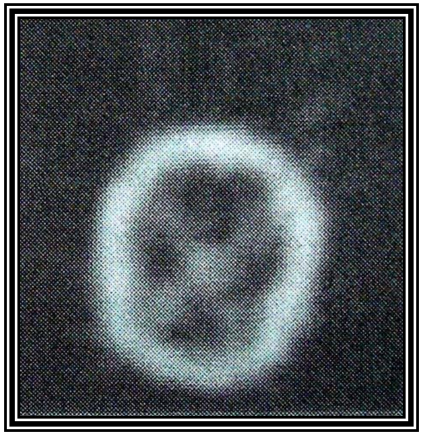


SPECT image of the brain

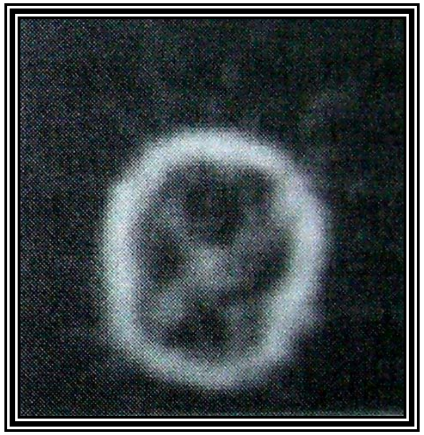
PRERECONSTRUCTION RESTORATION



Wiener

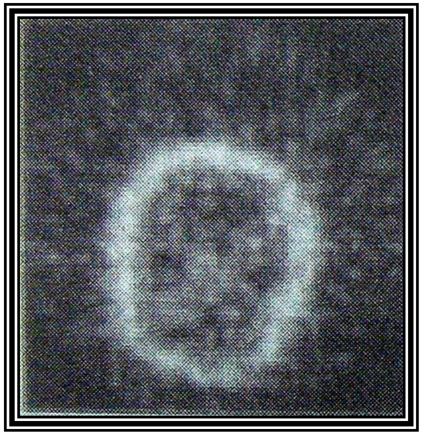


PSE



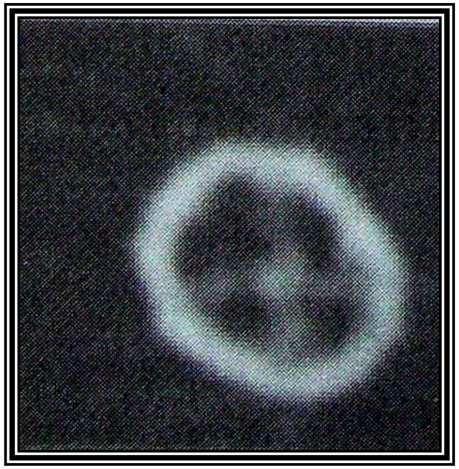
Metz

Example: SPECT images of the brain [3]

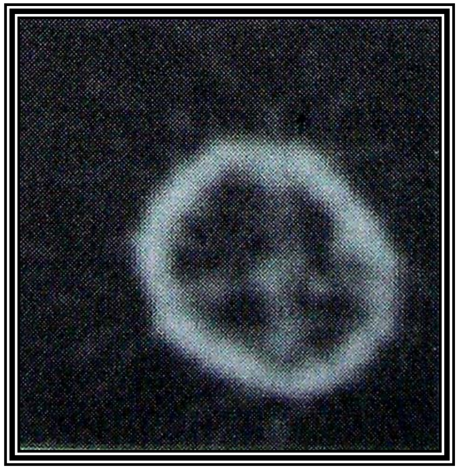


SPECT image of the brain

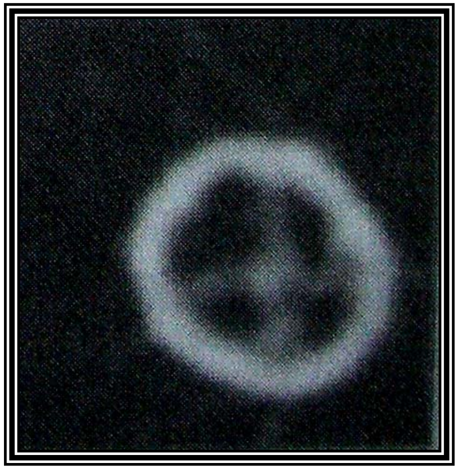
GEOMETRIC AVERAGING AND PRERECONSTRUCTION RESTORATION



Wiener



PSE



Metz

Conclusions

- Image degradation is present in even the most sophisticated and expensive imaging system
- Several techniques have been implemented to try to solve the problem
- Most of the restoration techniques require detailed and specific information about the original image and the degradation phenomena
- Several additional constraints may also be applied
- Always difficult to obtain the necessary accurate information
- The quality of the result depends on the quality of the information and of the constraints applied

Conclusions

A good solution of the problem is possible only with a good understanding of it

QUESTIONS?

Thank you!