Image Coding and Data Compression

Part 1 : Image Coding

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The Need for Coding and Compression

- Film technology is outdated and cumbersome.
- Medical Images are Digital.
- High Spatial Resolution and Fine grey level quantization are required for medical images.
- Volumetric data obtained by CT or MRI could be of size 512x512x64. At 16 bits/voxel, 32 MB of storage space is required.
- Digital Images may be compressed via image coding and data compression techniques.

Information Theory

- Entropy is a measure of uncertainty or randomness in an image.
- Higher entropy gives more information about the image.
- Higher Correlation between data

  ↓

  Higher Redundancy

  ↓

  Lesser Entropy

  ↓

  Lesser Information

Redundancy

- Code Redundancy
  - All pixel values do not occur with equal probability
- Spatial Redundancy
  - Adjacent pixels are correlated
- Psychovisual Redundancy
  - Precise numbers are not needed to observe important features in the image
- Reduce redundancy through coding for better compression of images.

Lossless vs Lossy Compression

- Lossless Coding
  - Original image can be recovered from the coded and compressed image without any loss in information.
- Lossy Coding
  - Original data cannot be recovered with complete numerical accuracy from the compressed image.
- Numerically lossy coding may be perceptually or diagnostically lossless.

Distortion Measure

- Let us define a error Image as

$$e(m,n) = g(m,n) - f(m,n)$$

- RMS value of the error

$$\sqrt{\frac{1}{N^2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\left[g(m,n)-f(m,n)\right]^2}$$

- SNR is defined as

$$\frac{\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} g^2(m,n)}{\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} e^2(m,n)}$$

Fundamental Concepts of Coding

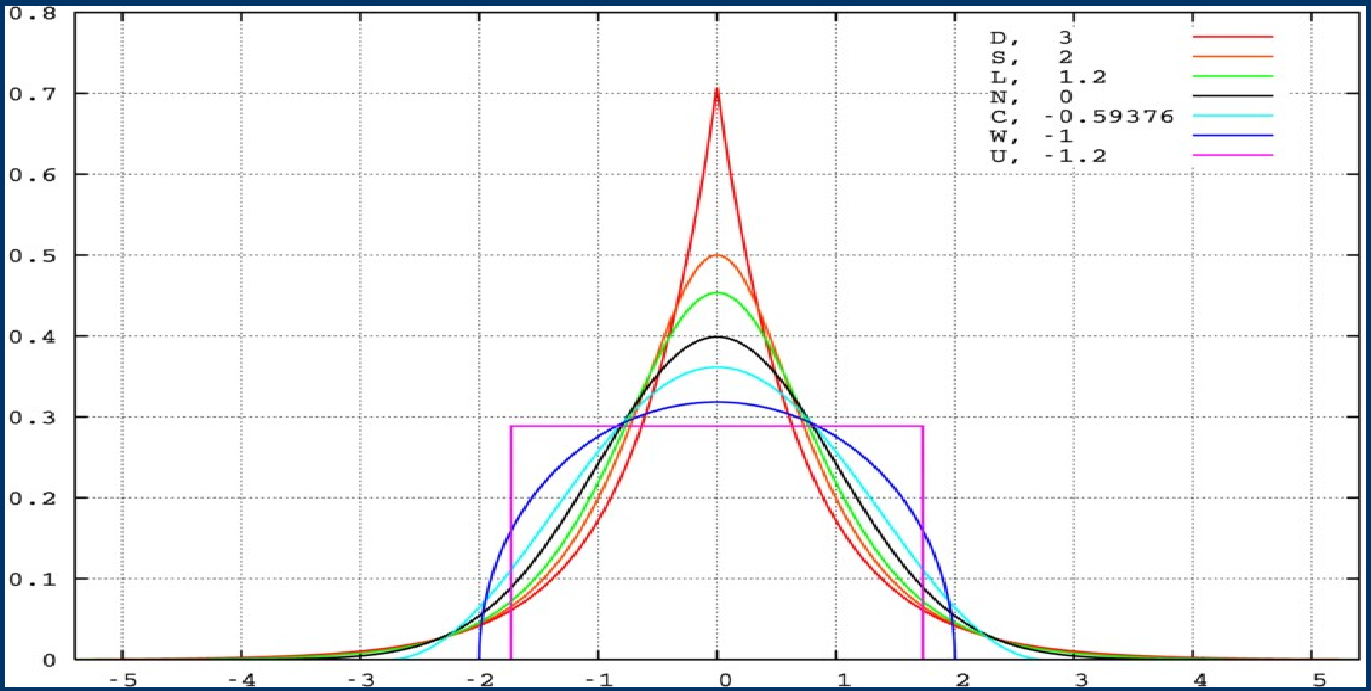
- Alphabet is a set of pre-defined symbols.
- Word is a finite sequence of symbols from an alphabet.
- Code is a mapping of words from a source alphabet into words of code alphabet.
- A code is said to be distinct if each code word is distinguishable from other code words.

Fundamental Concepts of Coding

- A distinct code is uniquely decodable if every code is identifiable in a sequence of code words.
- A uniquely decodable code must be decodable on a word to word basis.
- If no code word is a prefix of another, then it is also instantaneously decodable.
- A code is optimal if it is instantaneously decodable and has minimum average length for a given pdf.

Direct Source Coding

- Coding method is directly applied to the pixel values generated by the source.
- Patterns of limited variation and high correlation
- Usually images have non-uniform pdf.



Huffman Coding

- Some pixel values have higher probabilities than other pixel values.
- Use short code words for pixels with high probabilities of occurrence.
- Code words are variable in length.
- Average code word lengths are lesser than that provided by fixed length codes.
- Codes are uniquely and instantaneously decodable.

Huffman Coding

- Prior knowledge of pdf is assumed.
- Huffman code is optimal only for a given source pdf and has to be redesigned for changes in pdf.
- Huffman coding yields best results for highly non-uniform or concentrated pdf.
- Without prior decorrelation, the code word length is increasing for sources with several symbols.

Huffman Coding - Example

W	$P(w_i)$	C	V	$P(v_i)$		U	$P(u_i)$		X	$P(x_i)$	
w_1	0.5	1	v_1	0.5	1	u_1	0.5	1	x_1	0.5	0
w_2	0.2	01	v_2	0.2	01	u_2	0.3	00	x_2	0.5	1
w_3	0.15	001	v_3	0.15	000	u_3	0.2	01			
w_4	0.1	0000	v_4	0.15	001						
w_5	0.05	0001									

Run-length Coding

- Images with high levels of correlation contain strings of repeated values of the same grey level.
- $$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 3 & 2 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 4 & 6 \end{bmatrix}$$
- Run-length coding can be done as follows

  Row 1: (1,10),(2,1),(3,1),(2,2),(1,1),(2,1)

  Row 2: (0,1),(1,10),(2,2),(3,1),(4,1),(5,1)

  Row 3: (1,1),(0,3),(1,8),(2,2),(4,1),(6,1)

Run-length Coding

- Best suited for bi-level images.
- Images with fine details, intricate texture and high resolution quantization with large numbers of bits per pixel may actually lead to data expansion.
- Can be advantageously applied to bit planes of grey level and colour images.
- Errors in run length can cause severe degradation of the reconstructed image due to loss of pixel position.

Arithmetic Coding

- Represent a string of input symbols by their individual p_l and cumulative probabilities P_l .
- Source string is represented by code point C_k and an interval A_k .
- A new symbol is encoded as follows $A_{k+1} = A_k p_l$ and the new code point is defined as $C_{k+1} = C_k + A_k P_l$
- Each symbol need not have a unique code word as in Huffman coding.

Lempel-Ziv Coding

- Probabilities of source symbols not known a priori.
- Pass symbols from source into sub strings or words.
- Map the sub strings of variable length into uniquely decipherable codes of fixed length.
- Lempel- Ziv coding may be viewed as a search through a fixed size variable content dictionary for words that match the current string.
- Lempel-Ziv-Welch (LZW) Coding.

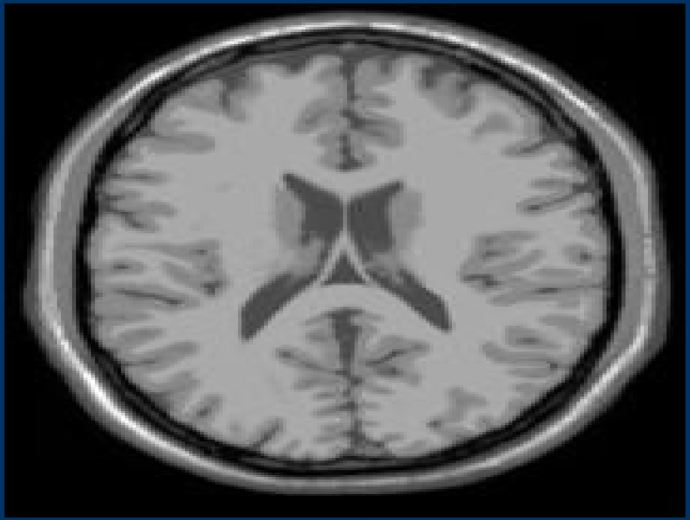
Contour Coding

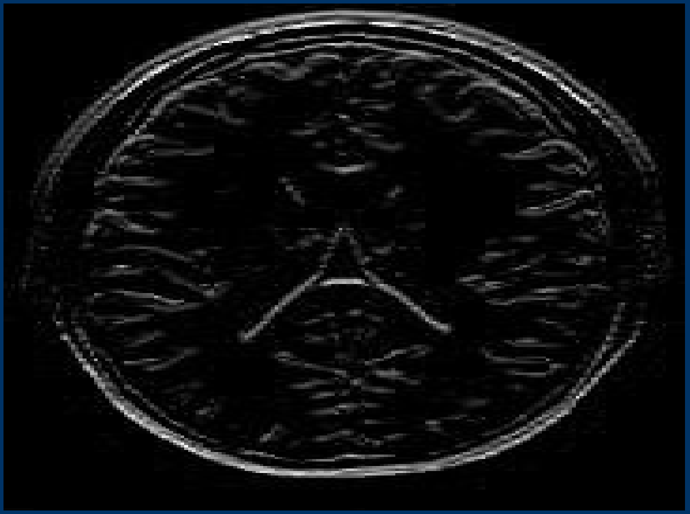
- Pixels of same grey levels occur in 2D contours or patterns in the image.
- Information related to all such contours may be used to encode the image.
- For each contour, encode co-ordinates of starting point, grey levels and sequence of steps needed to trace the contour.
- A consistent rule is required for repeatable tracing of contours.

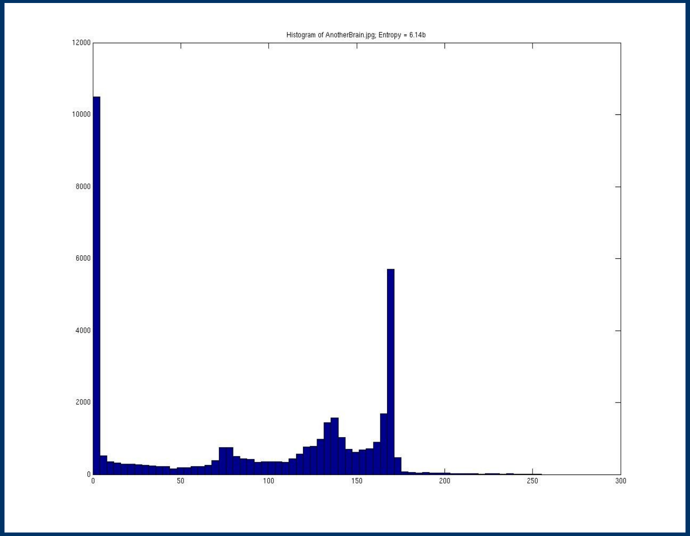
Decorrelation

- Decorrelation is a procedure to remove or reduce redundancy or correlation between pixels.
- Commonly used decorrelation techniques are
  - Differentiation
  - Transformation to another domain
  - Model based prediction
  - Interpolation
- Decorrelated data needs to be encoded and transmitted.
- Coding requirements are significantly reduced.

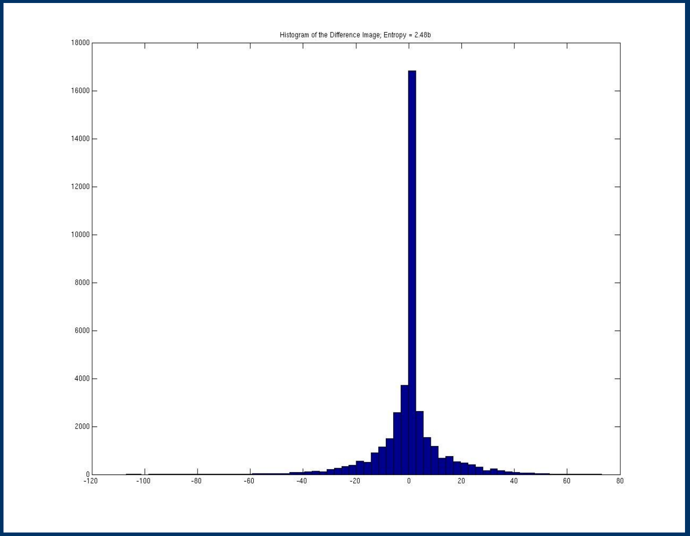
Decorrelation







Entropy = 6.14b



Entropy = 2.48b

Transform Coding

- Orthogonal Transforms - Rotation of co-ordinate system in signal space.
- Purpose of the transform – Decorrelation and Energy Concentration.
- We have already seen
  - DFT
  - WHT
- We will now see
  - Discrete Cosine Transform (DCT)
  - Karhunen Loeve Transform (KLT)

DCT

- DCT is a modification of DFT
- $$F(k,l) = \frac{a(k,l)}{N} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(m,n) \cos\left[\frac{\pi m}{N}(2k+1)\right] \cos\left[\frac{\pi n}{N}(2l+1)\right]$$

$$k = 0,1,2,\ldots\ldots,N-1$$

$$l = 0,1,2,\ldots\ldots,N-1$$

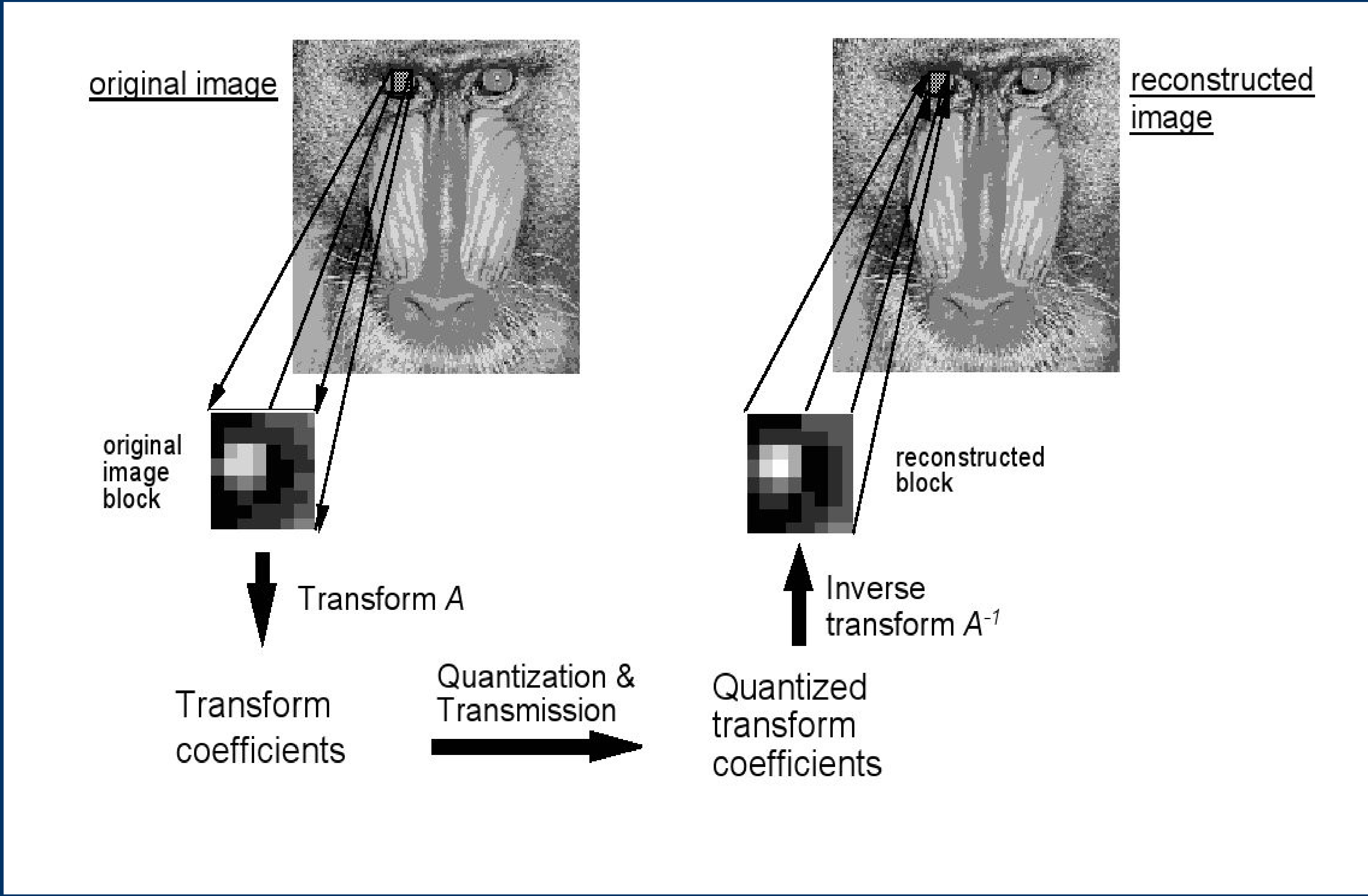
$$a(k,l) = \begin{cases} 1 & if\ (k,l)=(0,0) \\ \frac{1}{2} & otherwise \end{cases}$$

- Inverse DCT is given by

$$f(m,n) = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} a(k,l) F(k,l) \cos\left[\frac{\pi m}{N}(2k+1)\right] \cos\left[\frac{\pi n}{N}(2l+1)\right]$$

$$m = 0,1,2,\ldots\ldots,N-1 \qquad n = 0,1,2,\ldots\ldots,N-1$$

DCT

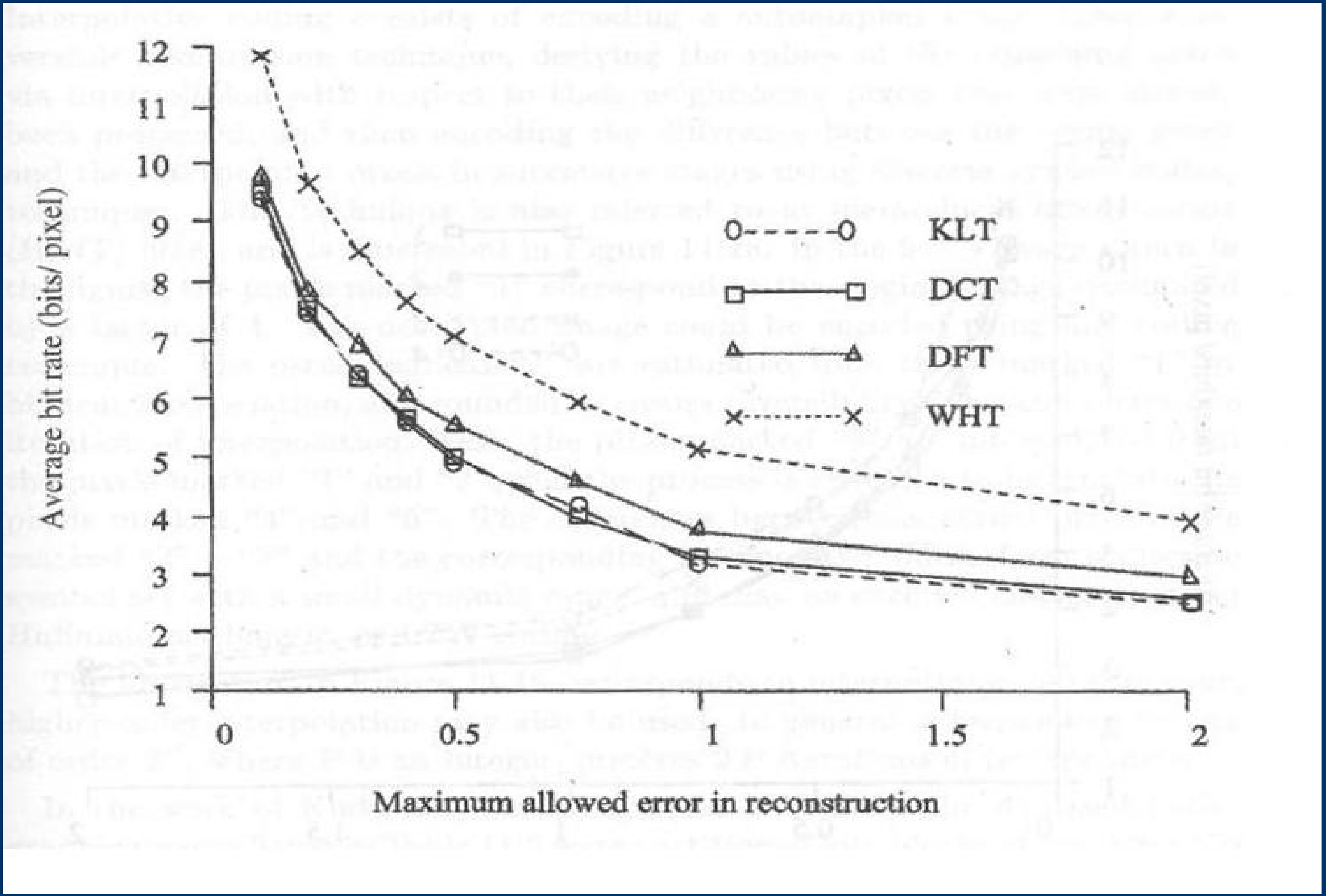


original image
reconstructed image
original image block
reconstructed block
Transform A
Inverse transform A-1
Transform coefficients
Quantization & Transmission
Quantized transform coefficients

KLT

- Principal Component Transform or Hotelling Transform or Eigenvector Transform.
- Based on statistical properties of the image.
- KLT yields decorrelated transform coefficients (Covariance matrix is diagonal).
- Basis functions are the eigenvectors of the input covariance matrix.
- KLT achieves optimum energy concentration.

Comparison of Transforms



Encoding Transform Coefficients

- Transform coefficients are quantized for encoding irrespective of the transform used.
- Quantization errors are introduced.
- Maximum error limit must be derived so that pixels exceeding this limit are encoded separately.
- Use variable length encoding and bit allocation depending on pdf of transform coefficients

Interpolative Coding

- Sub sample the image (Decimation).
- Interpolate the remaining pixels through some suitable interpolation techniques.
- Compute the differences between the actual and interpolated pixels.
- Encode the difference using discrete symbol coding techniques.
- Differences can modelled using Laplacian pdf.
- Interpolative coding of order 2^P

Predictive Coding

- Correlation between adjacent samples spatially or temporally – statistical redundancy.
- Sample of a signal may be predicted from previous samples.
- Linear Prediction Model for Images

$$\tilde{f}(m,n) = -\sum_{p}\sum_{q} a(p,q)\, f(m-p, n-q)$$

where a(p,q) are the 2D LP model coefficients.

- Prediction Error and MSE can be calculated as

$$e(m,n) = f(m,n) - \tilde{f}(m,n) \qquad \epsilon^2 = E\left[e^2(m,n)\right]$$

Predictive Coding

- Error image e(m,n) has more concentrated pdf which leads t better compression.
- Derive Prediction Coefficients – 2D normal or Yule-Walker equations.
- $$\phi_f(r,s)+\sum_p \sum_q a(p,q)\phi_f(r-p,s-q)=\begin{cases} 0 & (r,s)\in ROS \\ \epsilon^2 & (r,s)=(0,0) \end{cases}$$

  where ϕ_f is the ACF and ROS is the region of support.
- Prediction error can be encoded using any discrete symbol coding method.

Predictive Coding

- Multichannel Linear Prediction is a combination of 1D and 2D Linear Prediction.
- Certain number of rows of the image can be seen as a collection of multichannel signals.
- Prediction coefficient matrices can be calculated using Levinson-Wiggins-Robinson algorithm or directly from the image through Burg algorithm.
- For error free reconstruction at the decoder, the prediction coefficients have to be recomputed.

Predictive Coding

- Adaptive 2D recursive Least Squares Prediction.
- LP model with constant prediction coefficients assume stationarity of image generating processes.
- Assumption is hardly valid for real world images.
- To overcome this problem,
  - Block computation of prediction coefficients
  - Adapt coefficients recursively to changing statistical characteristics of the image
    - Minimize the weighted sum of prediction errors in the least squares sense.

Quick Recap

- Lossless data compression is desirable in medical images.
- Information theoretic considerations.
- Direct Source Coding
- Transform Coding
- Predictive Coding

Thank You

Any Questions..??