Exercise 4.3, page 359

Compute by hand the result of linear convolution of the following two images:

$$f(m,n) = \begin{bmatrix} 3 & 5 & 5 & 5 \\ 0 & 0 & 1 & 3 \\ 4 & 4 & 4 & 3 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{bmatrix} \quad \text{and} \quad h(m,n) = \begin{bmatrix} 3 & 5 & 1 \\ 4 & 2 & 3 \\ 1 & 3 & 2 \end{bmatrix}.$$

The convolution is defined as:

$$\begin{aligned} g(m,n) &= h(m,n) \star f(m,n) \\ &= \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} h(k,l) f(m-k, n-l) \end{aligned}$$

Lets assume that $f(m,n)=0$, when $m<0$ or $n<0$ or $m>4$ or $n>3$ (zero-padding).
Matrices were

$$f(m,n)=\begin{bmatrix}3&5&5&5\\0&0&1&3\\4&4&4&3\\2&2&2&2\\2&2&2&2\end{bmatrix} \quad \text{and} \quad h(m,n)=\begin{bmatrix}3&5&1\\4&2&3\\1&3&2\end{bmatrix}.$$

$\mathbf{g(0,0)}=3\cdot3+4\cdot0+1\cdot0+5\cdot0+2\cdot0+3\cdot0+1\cdot0+3\cdot0+2\cdot0=\mathbf{9}$.
$\mathbf{g(1,0)}=3\cdot0+4\cdot3+1\cdot0+5\cdot0+2\cdot0+3\cdot0+1\cdot0+3\cdot0+2\cdot0=\mathbf{12}$.
$\mathbf{g(2,2)}=3\cdot4+4\cdot1+1\cdot5+5\cdot4+2\cdot0+3\cdot5+1\cdot4+3\cdot0+2\cdot3=\mathbf{66}$.
$\mathbf{g(5,3)}=3\cdot0+4\cdot2+1\cdot2+5\cdot0+2\cdot2+3\cdot2+1\cdot0+3\cdot2+2\cdot2=\mathbf{30}$.

$$g(m,n) = \begin{bmatrix} \mathbf{9} & 30 & 43 & 45 & 30 & 5 \\ \mathbf{12} & 26 & 42 & 59 & 41 & 18 \\ 15 & 46 & \mathbf{66} & 77 & 53 & 22 \\ 22 & 40 & 55 & 56 & 41 & 17 \\ 18 & 44 & 60 & 59 & 39 & 14 \\ 10 & 20 & \mathbf{30} & 30 & 20 & 10 \\ 2 & 8 & 12 & 12 & 10 & 4 \end{bmatrix}.$$