T-61.140 Signal Processing Systems

Summer exam, Mon 13.6.2005 12-15, main building.
You are NOT allowed to use math reference book. **(Graphical) calculator is allowed, but extra memory has to be erased**. Formulas in accompaning paper. **Write down clear steps in your solutions. Begin** a new problem **from a new page.**

1) (6p) Multiple choises. Write down a table similar to that one below. Reply one option **A**, **B** or **C**, which is correct or closest. Right answer +1 p, wrong answer −0.5 p, or no answer 0 p. Reply to as many as you want. Minimum points 0, max 6. No explanations needed.

m1	m2	m3	m4	m5	m6	m7	m8	m9

m1) Notation $x(t)$ means normally [**A**] analog signal, [**B**] discrete-time, but continuous in amplitude, signal, [**C**] digital signal.

m2) Convolution is [**A**] multiplication (with respect to time) of two signals, analog or digital, [**B**] addition of two signals, [**C**] basic operation in signal processing, with which it is possible to get output of LTI-system, when input and impulse response are known.

m3) If $x[n] = \delta[n] + 2\delta[n-10] - \delta[n-20]$ is fed into LTI-system, then [**A**] output can be computed as a linear combination of shifted impulse responses, [**B**] the length of output is always 21 samples, [**C**] output is bounded and the filter is therefore always stable.

m4) FIR filter: [**A**] there are several y terms in difference equation, [**B**] always stable, [**C**] denominator polynomial in frequency response is of order 1.

m5) LTI filter, whose impulse response is $\{\underline{1}, -1, 1, -1, 1, -1, \ldots\}$ (notation $\underline{a}$ represents the sample at origo) [**A**] is stable, [**B**] is FIR filter, [**C**] has feedback.

m6) If the computation of the filter is recursive, then [**A**] the output can be computed using only the present and previous input values, [**B**] there are not feedback loops in the system, [**C**] the filter may be astable.

m7) The spectrum $|X(e^{j\omega})|$ can be achieved from the signal $x[n]$ [**A**] by convolution, [**B**] by Laplace transform, [**C**] by Fourier transform.

m8) Consider the filter $H(e^{j\omega}) = 1 - 0.1e^{-3j\omega}$. [**A**] Length of impulse response is three. [**B**] Length of impulse response is four. [**C**] The order of the filter is two.

m9) The signal $x(t)$ is sampled with sampling frequency f_s, and the length of the sequence $x[n]$ becomes 60000. If the sampling period T_s was doubled, what would be the length of the sequence $x[n]$? [**A**] 30000, [**B**] 60000, [**C**] 120000.

2) (6p) Examine discrete-time filters defined by difference equations

$$\begin{aligned} y_1[n] &= -x[n] + x[n-1] + 3x[n-2] \\ y_2[n] &= -2x[n+2] + x[n+1] \end{aligned}$$

a) Set filters parallel and compute impulse response $h_p[n]$.

b) Set filters in series (cascade) and compute impulse response $h_c[n]$ with convolution.

c) If the output of the cascade connection is

$$y_c[n] = 2\delta[n] - 3\delta[n-1] - 9\delta[n-2] + 9\delta[n-3] + 10\delta[n-4] - 6\delta[n-5]$$

what was the input $x[n]$?

d) Is the cascade system causal? Justify.

3) (3p) Examine the periodic sequence

$$x[n] = \sin(\pi n + \pi/4) - 2 \cdot \cos(2\pi n/3)$$

What is the fundamental period N_0?

4) (9p) The following program is reading the input sequence (`input_stream`) from A/D-converter, and does some numerical computation, and returns the output sequence back to D/A-converter (`output_stream`). Discrete-time filter is represented with pseudo code:

```
y0 := 0; y1 := 0; y2 := 0;
x0 := 0; x1 := 0; x2 := 0;
K := 1;  % init
while TRUE {
  x2 := x1; x1 := x0; y2 := y1; y1 := y0;
  x0 := K * read_next_item(input_stream);
  y0 := x0  + x2 - 0.64 * y2;
  write_item(output_stream, y0);
}
```

a) Write down the difference equation of the filter and draw the flow diagram (block diagram) using the notations used in the course.

b) Define the frequency response of the filter $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$.

c) Compute the missing values of frequency response $H(e^{j\omega})$ and amplitude response $|H(e^{j\omega})|$ in Table 1. Sketch the graph of the amplitude repsonse. Is the filter lowpass / highpass / bandpass / bandstop / all-pass? What is the order of the filter?

d) What is the meaning of coefficient `K`? Give it another reasonable value (other than 1) and justify your choice.

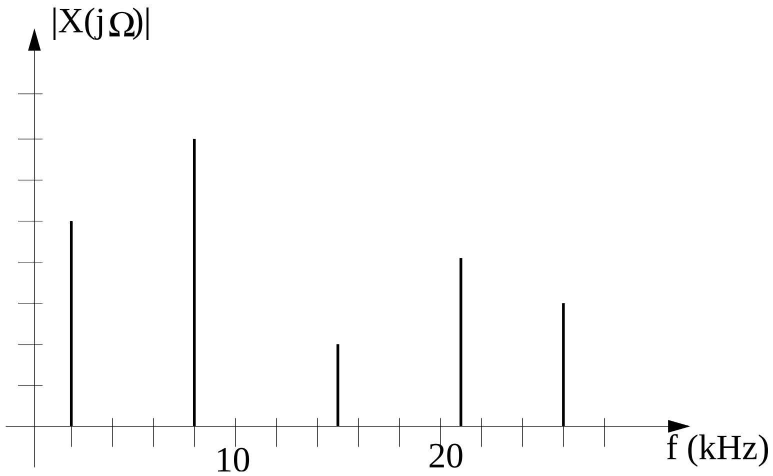
ω	$H(e^{j\omega})$	$\lvert H(e^{j\omega})\rvert$
0		
$\pi/2$		
π		

Taulukko 1: Problem 4, frequency and amplitude response of filter.

5) (6p) There is the spectrum $|X(j\Omega)|$ of signal $x(t)$ in Figure 1. Signal $x(t)$ can be expressed as a sum of cosine components

$$x(t) = K \cdot \big(\sum_i A_i \cdot \cos(2\pi f_i t + \theta_i) \big)$$

where K is a scaling constant and A_i, f_i and θ_i parameters for each cosine.



Kuva 1: Problem 5, spectrum $|X(j\Omega)|$.

a) Read the five pairs (f_i, A_i), $i = \{1, 2, 3, 4, 5\}$ from the figure, and compute signal value $x(1)$, when supposed that for each $\theta_i = 0$ and $K = 1$. Frequencies are multiples of thousands of Hertz.

b) Which of the following is/are true and why? **[A]** With sampling frequency $f_s = 30$ kHz there is no aliasing. **[B]** When sampling period $T_s > 2 \cdot 10^{-6}$ s there is no aliasing. **[C]** When sampling period $T_s < 2 \cdot 10^{-6}$ s there is no aliasing.

c) Sample the signal with $f_s = 20$ kHz. Sketch the spectrum $|X(e^{j\omega})|$ of sequence $x[n]$ in range $0 \ldots f_s/2$ Hz.

Have a nice summer!