Tik-61.140 Signal Processing Systems

2nd mid term exam / Exam, Tue 16.5.2000 9-12 C,D. UPDATED 16.5.2000. (Simula, Koskela, Parviainen)
You may use a mathematical handbook and graphical calculator. There are formulae on accompanying papers - use them!

2nd mid term exam: problems 3, 4, 5 and 6.
Exam: problems 1, 2, 4, 5 and 6.

Convolution: $y(t) = h(t) * x(t) = \int h(\tau)x(t-\tau)d\tau$, $y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$.

Euler's formula: $e^{j\omega} = cos(\omega) + j\, sin(\omega)$

Notation: impulse response $h(t)$ or $h[n]$, frequency response $H(j\omega)$ or $H(e^{j\omega})$, unit impulse $\delta(t)$ or $\delta[n]$, unit step $u(t)$ or $u[n]$, input x, output y.

1. (6p, exam) Consider the following three discrete-time systems

$$\begin{aligned} y_1[n] &= x[-n] + x[-n+1] - 1 \\ h_2[n] &= (-0.99)^n\, u[n+1] \\ y_3[n] &= y_3[n-2] + x[n] \end{aligned}$$

For each system, answer and explain briefly, if it is

a) linear
b) time-invariant
c) stable
d) causal

2. (6p, exam) Consider signals

$$\begin{aligned} x_1[n] &= \sin(\frac{31}{4}n) \\ x_2(t) &= 2\cos(\frac{27}{4}t - \pi/8) \\ x_3[n] &= \sum_{k=-\infty}^{\infty} \{\delta[n-3k-1] + \delta[n-3k-2]\} \end{aligned}$$

a) Examine, which of the signals are periodic. Calculate the basic period N_0 or T_0 of periodic signals.
b) For the all periodic signals in a), find the basic angular frequency ω_0, Fourier-series representation and Fourier-coefficients.

3. (6p, mid term exam) Answer, if the statement is false (F) or right (R). (a 1 p).

a) Discrete-time Fourier-transform is periodic with periods of half of sampling frequency.
b) Frequency response $H(e^{j\omega}) = e^{-j3\omega}$ attenuates (decrease in amplitude) high frequencies.
c) If $|H_l(j\omega)|$ is a low pass filter, whose values are scaled in range 0..1, then $|H(j\omega)| = 1 - |H_l(j\omega)|$ is a high pass filter.
d) In a nonlinear-phase LTI filter the frequencies of the input signal are changed.
e) Signal $x[n] = \sin(\theta n)/(\pi n)$ can be represented with sinc-function: $x[n] = (\theta/\pi)\,\text{sinc}(\theta n/\pi)$.
f) Fourier-coefficients of periodic discrete signal $\sin(\omega_0 n)$ are $a_{-1} = -1/(2j)$ and $a_1 = 1/(2j)$ and elsewhere $a_k = 0$.

4. (6p, exam/mid term exam) Edges are often searched in image processing. The image consists of pixels which have grey-scale values. Our filter scans the image row-wise sliding from left to right. The new, filtered pixel is a dot product of three adjacent pixels and a vector $\{-1,\ 2,\ -1\}$ so that the filter is causal. There are only read-only operations in the image to be filtered. Suppose that pixels outside the image are zero.

   a) Draw the block diagram of the filter (like in the figure 1), where $x[n]$ is the original pixel queue and $y[n]$ is the filtered pixel queue.
   b) Find $h[n]$ of the filter.
   c) Find the frequency response of the system $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$. Hint: use transform property $x[n-n_0] = e^{-j\omega n_0}X(e^{j\omega})$.
   d) Sketch $|H(e^{j\omega})|$ in range $0..\pi$ by calculating values, when ω gets values $\{0,\ \pi/4,\ \pi/2,\ 3\pi/4,\ \pi\}$. The filter is second-order, low order, so it behaves "smoothly" between the points. Hint: if calculated by hands: $e^{j\pi/4} \approx 0.7 + 0.7j$.
   e) Calculate the result of the convolution $y[n] = x[n] * h[n]$ when $x[n] = \{0,\ 0,\ 7,\ 8,\ 8,\ 8,\ 9,\ 1,\ 0,\ 0\}$.

5. (6p, exam/mid term exam) Consider a discrete-time system, whose block diagram is in the figure 1.

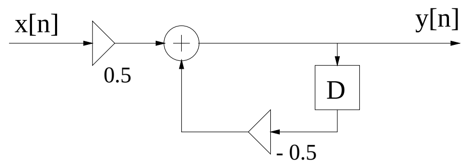

Figure 1: Block diagram for problem 5

   a) Find frequency response of the system $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$ (like in problem 4c).
   b) Sketch the amplitude response $|H(e^{j\omega})|$ (like in problem 4d).
   c) Find $h[n]$ by inverse transform, by solving the difference equation or by calculating values.
   d) Is the filter low pass, high pass, band pass, band stop or all-pass (all frequencies amplified by 1)?

6. (6p, exam/mid term exam) Consider the voice of telephone button 1, which consists of the sum of two cosine signals (of form $\cos(2\pi f_k t)$), $f_1 = 700$ Hz, $f_2 = 1200$ Hz. There is a time domain plot of the signal in the figure 2.

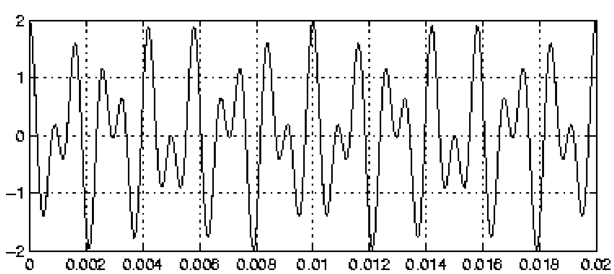

Figure 2: Button "1" in time domain

   a) Draw the spectrum (absolut value of Fourier-transform) of the sampled, discrete signal, in range $0..0.5\,f_s$, where the sampling frequency f_s is
      i) 950 Hz
      ii) 1900 Hz
      iii) 3800 Hz
   b) The voice of button "2" consists of cosine signals whose frequencies are 700 and 1340 Hz. Let the sampling frequency be 3800 Hz. There are two ideal filters available

      S_1 : low pass filter, let frequencies below 1000 Hz flow.
      S_2 : low pass filter, let frequencies below 1300 Hz flow.

      The button voices "1" and "2" are played in arbitrary order. Design a system which lets only the cosine component 1200 Hz flow through by using one or more ideal low pass filters available. Draw the chart of the system in frequency domain (S_1, S_2, ...).