Tik-61.140 Signal Processing Systems

2nd mid term exam / Exam, Thu 17.5.2001 9-12 T1. (Simula, Koskela, Parviainen)
You may use a mathematical handbook and graphical calculator. There are formulae on accompanying papers - use them!

Problems:
2nd mid term exam: 3, 4, 5 and 6.
Exam: 1, 2, 4, 5 and 6.

1. (6p, exam) Properties of systems and signals

   a) Discrete system is defined by difference equation $y[n] = x[n+1] - 2x[n] + x[n-1]$. Calculate, if the system is linear. Calculate, if the system is time-invariant.

   b) LTI-system is defined by impulse response $h[n] = \left(\frac{1}{3}\right)^n u[n+1]$. Is the system stable? Is the system causal?

   c) Consider a discrete sequence $x[n] = cos(\frac{\pi}{4}n) + 2\,sin(\frac{\pi}{6}n + \frac{\pi}{8})$. Is $x[n]$ periodic? If it is, what is the basic period?

2. (6p, exam) Consider a discrete-time LTI-system in the figure below. It consists of two components, which are connected as seen in (b). The impulse response of subsystem h_1 is $h_1[n] = \delta[n] - \delta[n-1]$. $h_2[n]$ is unknown. Using input $x[n]$ from (a), output $y[n]$ from (c) is received.

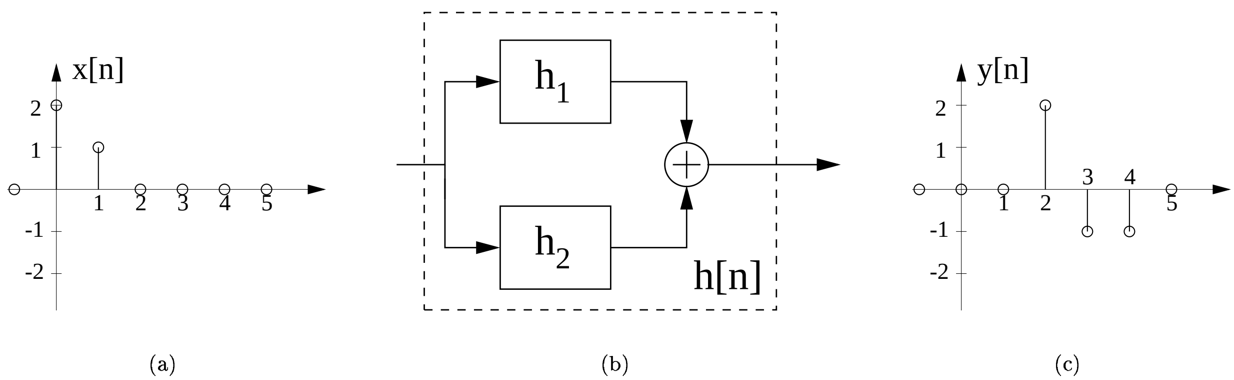

Figure 1: Figures of Problem 2: (a) Input $x[n]$, $x[n] = 0$, when $n < 0,\ n > 1$, (b) LTI-system, (c) Output $y[n]$, $y[n] = 0$, when $n < 2,\ n > 4,\ n \in Z$.

   a) What is the length of $h_2[n]$. Explain.

   b) Define the impulse response $h_2[n]$ of the subsystem.

   c) What is the output $y[n]$, if the input is $x[n] = \delta[n]$?

3. (6p, mid term exam) Answer, if the statement is true (T) or false (F). Correct answer +1p, wrong -1p, no answer 0p.
   ATT! There are seven statements, max points 6.

   a) It is known that changes in the signal can be found with a filter whose impulse response is $h[n] = -\delta[n] + 2\delta[n-1] - \delta[n-2]$. Claim: The type of filter is highpass.

   b) If the spectrum of output is $Y(e^{j\omega}) = e^{-j\omega}\frac{0.5}{1-0.5\,e^{-j\omega}}$, then $y[n]$ gets value 0.5 when $n = 0$.

   c) The bigger the cut-off frequency of a lowpass filter, the shorter rise time of step function.

   d) Group delay of a linear-phase filter is zero.

   e) Consider a voice signal, which is sampled for the computer with 8192 Hz as the sampling frequency. The signal component of 9000 Hz is aliased. Claim: When signal is reconstructed back to analogue, one can hear a component of 7384 Hz. Phase information can be ignored.

   f) Discrete-time Fourier-transform is periodic with 2π.

   g) Inverse Fourier-transform $x[n]$ of complex valued spectrum $X(e^{j\omega})$ can be real (complex part is zero).

4. (6p, exam, mid term exam) A causal and stable LTI-system is defined by difference equation:

$$y[n] - 1.7\,y[n-1] + 0.72\,y[n-2] = x[n]$$

a) Draw a block diagram of the system.

b) Define frequency response $H(e^{j\omega})$.

c) It is known that the filter is lowpass. Define scaling factor a so that the maximum value of amplitude spectrum of filter is one.

d) Calculate impulse response $h[n]$ in closed form using the scaled filter.

5. (6p, exam, mid term exam) See Figure 2 where is a sum of two analogue periodic cosine signals $x(t) = cos(2\pi f_1 t + \theta_1) + cos(2\pi f_2 t + \theta_2)$. The time axis is between $-0.05..0.3$ seconds.

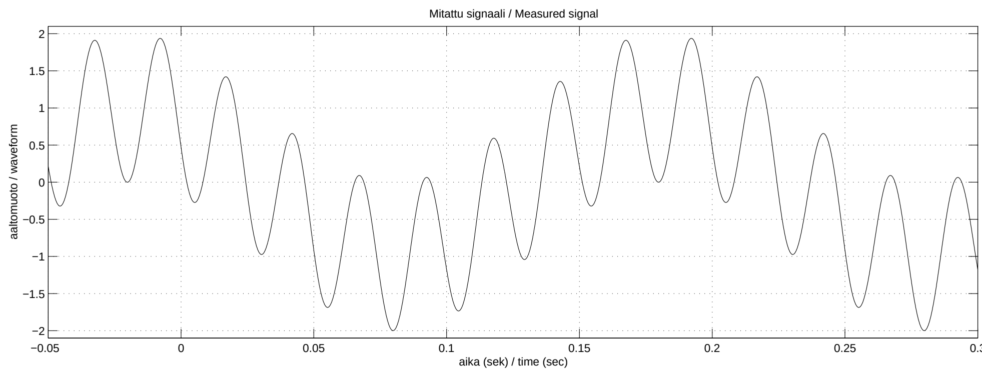

Figure 2: The sum signal of Problem 5

a) Draw the amplitude spectrum $|X(j\omega)|$ of continuous-time signal $x(t)$ between 0..50 Hz. Hint: one cosine corresponds to one peak in spectrum.

b) Let the sampling frequency be 20 Hz. Draw the amplitude spectrum $|X(e^{j\omega})|$ of sampled sequence $x[n]$ between 0..10 Hz.

c) Reconstruct a continuous signal $\hat{x}(t)$ using samples from b). Sketch the signal in time domain between 0..0.2 seconds. What are the differences compared to original signal $x(t)$?

6. (6p, exam, mid term exam) Answer with a few sentences.

a) What does amplitude spectrum (or amplitude response) of signal show?

b) What does phase response (or phase spectrum) show? Give an example of two signals in time domain which have same amplitude spectrum but different phase spectrum.

c) Your job is (i) to remove sudden, short rustles/scratches from a voice file, (ii) remove a continuous disturbance of possible supply current. Do you start to work these cases in time or frequency domain? Why?

d) Discrete (voice/image) signal is Fourier-transformed. In what way can you compress the signal using frequency representation? Consider, for example, a portrait of a person in JPEG-image format.