T-61.140 Signal Processing Systems

2nd mid term exam / final exam, Mon 16.5.2005 13-16, hall A.
You are NOT allowed to use any math reference book. **(Graphical) calculator allowed, erase extra memory**. Formula table with accompanying sheet. **Show clearly all steps in your results. Begin** a new problem **from a new page.**

If you did a mid term exam on 4.5.2005, you cannot do it again today. If you did a final exam on 4.5.2005, you cannot do it again today.

IF you do mid term exam, REPLY to 3, 4, 5, 6 and 7.
IF you do final exam, REPLY to 1, 2, 4, 5, 6 and 7.
Write down, if you are doing **mid term exam OR final exam.**

1) (6p, final exam)

   a) What is the fundamental period N_0 of the sequence $x[n] = 3\cos((\pi/3)n) + \sin((\pi/4)n - \pi/3)$.
   b) Is the discrete-time filter $y[n] = (x[n])^2 + 0.5x[n-1]$ linear? Is it time-invariant? Compute or explain.
   c) The impulse response of the LTI filter is $h[n] = (-1/2)^n u[n-2]$. Is it causal? Is it stable? Show using the definitions.

2) (6p, final exam) Examine discrete-time filters defined by difference equations

$$\begin{aligned} y_1[n] &= x[n] - x[n-1] + 3x[n-2] \\ y_2[n] &= x[n-1] - 2x[n-2] \end{aligned}$$

   a) Set the filters parallel and compute impulse response $h_p[n]$.
   b) Set the filters in cascade (series) and compute impulse response $h_c[n]$.
   c) If the output of the cascade connection is $y_c[n] = -2\delta[n+1] + 7\delta[n] - 13\delta[n-1] + 17\delta[n-2] - 6\delta[n-3]$, what was the input $x[n]$?

3) (6p, mid term exam) Statements. If you think that the statement is true, write T. If you think that the statement is false, write F. Each correct answer +1 p, wrong answer -0.25 p, or no answer 0 p. You don't have to explain your results.

   a) Fourier transform of the sequence $x[n] = (0.5)^{n-2}u[n]$ is $X(e^{j\omega}) = 0.25/[1 - 0.5e^{-2j\omega}]$.
   b) The order of the filter $H(e^{j\omega}) = \left(1 - 0.1e^{-2j\omega}\right)^2$ is four.
   c) The filter $H(e^{j\omega}) = 2e^{-j\omega} + e^{-2j\omega} + 0.5e^{-3j\omega} + 0.25e^{-4j\omega} + \ldots$ has the corresponding difference equation $y[n] = 2x[n-1] + 0.5y[n-1]$.
   d) The phase response of the filter $H(e^{j\omega}) = 1 - e^{-2j\omega}$ is linear.
   e) Signals $x_1(t) = \cos(2\pi t)$ and $x_2(t) = \sin(2\pi t)$ have the same amplitude spectrum $|X_1(j\Omega)| \equiv |X_2(j\Omega)|$.
   f) The phase spectrum $\angle X(e^{j\omega})$ of the real-valued sequence $x[n]$ is periodic with 2π period.

4) (6p, mid term exam, final exam) Consider a discrete-time filter given below.

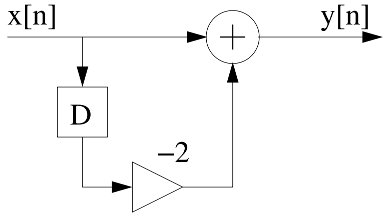

Figure 1: The flow (block) diagram of LTI system.

a) What is the difference equation?
b) What is the impulse response $h[n]$?
c) Determine the frequency response $H(e^{j\omega})$ of the filter and sketch the amplitude response $|H(e^{j\omega})|$. Is the filter lowpass, highpass, bandpass, bandstop or all-pass filter?
d) Replace each delay of filter by double delays and sketch the amplitude response again.

5) (6p, midterm exam, final exam) Consider a continuous-time signal $x(t)$, which consists of three cosine components (150 Hz, 350 Hz, 450 Hz)

$$x(t) = \cos(2\pi \cdot 150 \cdot t) + 2 \cdot \cos(2\pi \cdot 350 \cdot t) + 3 \cdot \cos(2\pi \cdot 450 \cdot t)$$

a) Sketch the amplitude spectrum $|X(j\Omega)|$ of the continuous-time signal in $0 \ldots 1500$ Hz.
b) What is the biggest sampling interval T_s (in seconds), where no aliasing occurs?
c) Demonstrate the effect of the too small sampling frequency in frequency domain, when the sampling frequency is $f_s = 380$ Hz. Draw the spectrum $|X(e^{j\omega})|$ of the discrete-time signal in range $0 \ldots 190$ Hz.

6) (6p, mid term exam, final exam) You should produce a sequence of squares $\{0,\ 1,\ 4,\ 9,\ 16,\ 25,\ \ldots\}$ using a third order recursive LTI filter. Figure out or compute the coefficients of the filter and determine the initial values of delay registers. Hint: Think, how a new sample of the sequence can be represented using the previous values. Keep the input always as zero ($x[n] \equiv 0$, for all n).

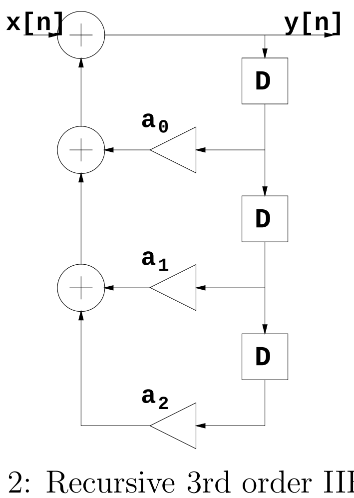

Figure 2: Recursive 3rd order IIR filter.

7) (2p mid term exam, 1p final exam) If you do not understand Finnish and therefore cannot fill in the feedback questionnaire in `http://www.cs.hut.fi/Opinnot/Palaute/kurssipalaute.html`, you can use the course feedback system of CIS lab: `http://www.cis.hut.fi/teaching/feedback.shtml`. The feedback is anonymous, however, in order to get points, you should give your student ID as a comment.