EFFICIENCY OF SIMPLE SHAPE DESCRIPTORS

M. PEURA, J. IIVARINEN
Helsinki University of Technology
Laboratory of Computer and Information Science
P.O. Box 2200 FIN-02015 HUT, Finland

Characterizing objects by their shape is a critical part in many computer vision applications. Several theoretically interesting approaches exist which, however, remain computationally too expensive. The goal of this paper is to emphasize the value of primitive shape descriptors. A set of simple descriptors is used as a feature vector in order to group similar objects together. Each descriptor is discussed separately and discriminatory power of their combination is demonstrated on irregular sample objects. The advantages of using simple shape descriptors are obvious: faster calculation and more general applicability. Sometimes "general applicability" implies poor performance in a specific application. A neural network method (Self-Organizing Map) is used in adapting a system to an application-specific shape space. The results of the experiments are good. It is shown that fairly simple shape descriptors can be flexibly used in complex recognition tasks involving irregular objects.

1 Introduction

Shape descriptors have been actively studied [7, 11] as an alternative to the approaches involving texture and colour distribution. Intuitively, using only the contour of an object instead of the whole area would imply a square-root proportional saving in calculations. This is practically true although there does not always exist analog counterparts between 1D and 2D operations. For example, one can hardly adapt textural features to contours whereas orientation of an object can be derived using either area or contour.

The underlying practical justification for the descriptors presented in this paper is the fact that a significant amount of information is contained in the contour of an object. In practical situations, variation of contours produced by 3D objects cause severe problems. Moreover, occlusion of contours due to overlapping objects may turn shape descriptors worthless. In this paper, contours of objects are assumed to be given in a representative pose. In other words, practical problems in image acquisition such as illumination and viewing angle are assumed to be solved.

Shape descriptors that use the contour of an object include the following techniques. Moment-based techniques have been used in object recognition since 1962 [4]. Moments derived from the contour of an object were used by Dubani et al. [1], and Gupta and Srinath [3]. Zahn and Roskies [12] used the

Fourier coefficients of a contour as shape descriptors. The chord distribution of a contour was proposed by Smith and Jain [10]. A scale-space technique to form a description for plane curves was proposed by Mokhtarian and Mackworth [9]. Evans et al. [2] proposed pairwise geometric histograms as shape descriptors. Iivarinen and Visa [5] proposed chain code histograms for irregular shape recognition.

We claim that one often applies too sophisticated methods in practical problems. For instance, why use Fourier descriptors unless objects really originate from a periodic process? We propose that one should hold to basic shape descriptors whenever possible. Our suggestion is based on the following simple aspects.

It is desired to find a method that is both simple and generally applicable. This truism is sometimes forgotten. For example, increasing computational resources allow us to see a planar contour of N pixels as a 2N-dimensional vector. Certainly, this does not imply that the phenomenon producing that contour be that complex.

Each descriptor should be semantically simple. It is safe to start designing descriptors by considering how human beings would describe objects. Some transformations, though motivated by mathematical properties such as reversibility, contradict this aim. Primitive descriptors such as *compactness* and *elongatedness* are revised in Sec. 2.

Combining descriptors should introduce a new perspective. It is expected that combinations of descriptors will provide new, more complex but still explicable descriptions. For example, if an object is nearly circular but not compact it must be ragged or star-like.

Some correlation between descriptors is acceptable. Clearly, descriptors are designed to detect possibly *different* properties. However, for a particular limited data set two descriptors might essentially measure the same quantity. For example, if all the contours in a data set are relatively simple and *smooth*, values of elongation and compactness will strongly correlate. On the other hand, no derived properties mentioned in above would appear if elementary descriptors were strictly orthogonal.

A general approach might need adjustment in applications. A set of measurements produces a vector, which is an instance of the current "application space". Redundancy within the elements can be removed by standard methods such as principal component analysis. We propose *Self-Organizing Map* (Sec. 3) because of its feasibility for visualization.

2 Simple shape descriptors

In this section, five simple shape descriptors are revised (Fig. 1). Variations of most of them have been widely used in object recognition [11]. The notations used here on are listed in Table 1.

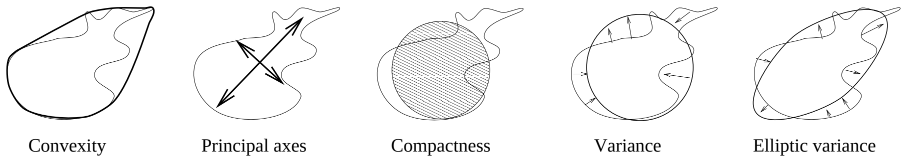

Figure 1: Five simple shape descriptors.

$\boldsymbol{p}_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$	contour point
$\boldsymbol{p} = \{\boldsymbol{p}_i\}, i = 1 \ldots N$	contour
$\mathcal{A}$	area
$\mathcal{P}$	perimeter
$\|\| \cdot \|\|$	vector length
$\boldsymbol{\mu} = \frac{1}{N} \sum_i \boldsymbol{p}_i$	centroid
$\mu_r = \frac{1}{N} \sum_i \|\|\boldsymbol{p}_i - \boldsymbol{\mu}\|\|$	mean radius
$\boldsymbol{C} = \frac{1}{N} \sum_i (\boldsymbol{p}_i - \boldsymbol{\mu})(\boldsymbol{p}_i - \boldsymbol{\mu})^\top$	covariance matrix
λ, $\exists \boldsymbol{e} : \boldsymbol{C}\boldsymbol{e} = \lambda \boldsymbol{e}$	eigenvalue of $\boldsymbol{C}$
$\boldsymbol{e}$	eigenvector of $\boldsymbol{C}$

Table 1: Notations.

2.1 Convexity

A convex hull is the minimal convex covering of an object. It can be thought as an elastic ribbon stretched around the contour of an object. A straightforward measure for convexity can thus be defined as the ratio of perimeters of the convex hull and the original contour,

$$conv = \frac{\mathcal{P}_{convexhull}}{\mathcal{P}}. \tag{1}$$

The algorithm for constructing a convex hull involves traversing the contour and minimizing turn angle in each step. The straightforward implementation of *conv* requires $\mathcal{O}(N^2)$ calculation steps. Algorithms of linear complexity has been reported as well [8].

2.2 Ratio of principal axes

Principal axes of a given object can be uniquely defined as the segments of lines that cross each other orthogonally in the centroid of the object and represent the directions with zero cross-correlation. This way, a contour is seen as an instance from a statistical distribution.

The lengths of the principal axes equal the eigenvalues $\lambda_{1,2}$ of the covariance matrix $\boldsymbol{C}$ of a contour (Table 1). Let us denote the elements of $\boldsymbol{C}$ as

$$\boldsymbol{C} = \begin{pmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{pmatrix}. \tag{2}$$

The ratio of principal axes is a reasonable measure of elongation and can be calculated directly:

$$prax = \frac{c_{yy} + c_{xx} - \sqrt{(c_{yy} + c_{xx})^2 - 4(c_{xx}c_{yy} - c_{xy}^2)}}{c_{yy} + c_{xx} + \sqrt{(c_{yy} + c_{xx})^2 - 4(c_{xx}c_{yy} - c_{xy}^2)}}. \tag{3}$$

The calculation of actual eigenvectors $\boldsymbol{e}_{1,2}$ or eigenvalues $\lambda_{1,2}$ remain unnecessary – they could be calculated with a further effort equal to that of *prax* itself. Thus, the computational effort for *prax* is small – it requires only $\mathcal{O}(N)$ steps.

2.3 Compactness

Compactness is often defined as the ratio of the squared perimeter and the area of an object. It reaches the minimum $\frac{(2\pi r)^2}{\pi r^2} = 2\pi$ in a circular object and approaches infinity in thin, complex objects. The measure applied in this paper is the ratio of the perimeter of a circle with equal area as the original object and the original perimeter, i.e.,

$$comp = \frac{\mathcal{P}_{circle}}{\mathcal{P}} = \frac{2\sqrt{\mathcal{A}\pi}}{\mathcal{P}}, \ \mathcal{A}_{circle} = \mathcal{A}. \tag{4}$$

The calculation of the area $\mathcal{A}$ requires a single traversal of the contour [11]. The same applies for the contour length $\mathcal{P}$ and consequently, *comp* can be calculated in $\mathcal{O}(N)$ operations.

2.4 Circular variance

Sometimes a shape should be compared against a template. Clearly, a circle must be one of the simplest and most general possible choices. The proportional mean-squared error with respect to solid circle, or circular variance, can be defined

$$cvar = \frac{1}{N\mu_r^2}\sum_i (||\boldsymbol{p}_i - \boldsymbol{\mu}|| - \mu_r)^2. \tag{5}$$

Thus, *cvar* gives zero for a perfect circle and increases along shape complexity and elongation. *cvar* requires $\mathcal{O}(N)$ calculation steps.

2.5 Elliptic variance

Starting from *cvar*, a natural choice to increase the degrees of freedom is to allow elongation, i.e., to fit an ellipse and measure the mapping error. Intuitively, it seems logical to find an ellipse having an equal covariance matrix, $C_{ellipse} = C$. It is practically effective to apply the inverse approach yielding

$$evar = \frac{1}{N\mu_{r_C}}\sum_i \left(\sqrt{(\boldsymbol{p}_i - \boldsymbol{\mu})^\top \boldsymbol{C}^{-1}(\boldsymbol{p}_i - \boldsymbol{\mu})} - \mu_{r_C}\right)^2, \tag{6}$$

$$\mu_{r_C} = \frac{1}{N}\sum_i \sqrt{(\boldsymbol{p}_i - \boldsymbol{\mu})^\top \boldsymbol{C}^{-1}(\boldsymbol{p}_i - \boldsymbol{\mu})}$$

Despite the apparent complexity *evar* requires $\mathcal{O}(N)$ calculation steps.

3 Experiments

The shape descriptors listed in the previous section were applied to a set of irregular shapes. More specifically, a five-element *feature vector* was extracted for each sample. The sample set consisted of 79 contours originating from an industrial quality control process. The covariance matrix for the descriptors is given in Table 2.

As may be expected, *conv* and *comp* as well as *prax* and *cvar* have high covariances. On the other hand, *conv* and *prax* seem to be independent properties in involved samples. It should be noticed that the given covariance values apply only to this particular set of samples. Further considerations are presented in the discussion.

The feature vectors were further used to train the Self-Organizing Map [6]. A result is shown in Fig. 2. The consistent and continuos organization of the shapes meets the eye. Circularity dominates the lower left corner of the

	conv	*prax*	*comp*	*cvar*	*evar*
conv	100	0	83	-10	-35
prax	0	100	45	-85	-39
comp	83	45	100	-58	-63
cvar	-10	-85	-58	100	73
evar	-35	-39	-63	73	100

Table 2: Scaled covariance matrix of the descriptors applied to a sample set.

map and elongated shapes the upper right corner. Moreover, smooth contours tend to be at the left and rough ones at right hand side of the figure. The transformations between different types of shapes are smooth which in fact is an essential property of Self-Organizing Maps. The banana-like shapes should be noticed to the right of the center in Fig. 2: none of the elementary descriptors alone detects this property but their combination does.

4 Discussion

There is a direct and intuitive trade-off in developing faster and simpler feature descriptors. Many proposed shape descriptors rely on the assumption that statistical measures, no matter how cryptic they might be, produce similar values for similar objects. However, one should pay attention to possible loss of information when applying descriptors with unclear semantic meaning: a descriptor might perform many-to-one mapping, thus failing in a classification task. Counter-examples exist as well. Hu showed that an infinite set of moments can be used to reconstruct an image [4]. It is, however, difficult to find verbal descriptions for high-level moments involved.

The descriptors listed in this paper have each a comprehensible explanation and can be applied separately in practical applications. Consequently, the authors studied visually the distributions of all the 10 possible descriptor pairs. It seems probable that three or even two of the reviewed descriptors will suffice to obtain reasonable classification performance. However, this assumption applies only for this data set — the distribution of values is inevitably case-dependent.

The Self-Organizing Map is an ideal tool in exploring structure of high-dimensional data. It produces a non-linear topology-preserving mapping from a data space to the two-dimensional space. This means that the Self-Organizing Map resolves the internal dimensionality of data. For example, the contours in Fig. 2, which were ordered by SOM according to five descriptors, seem to have

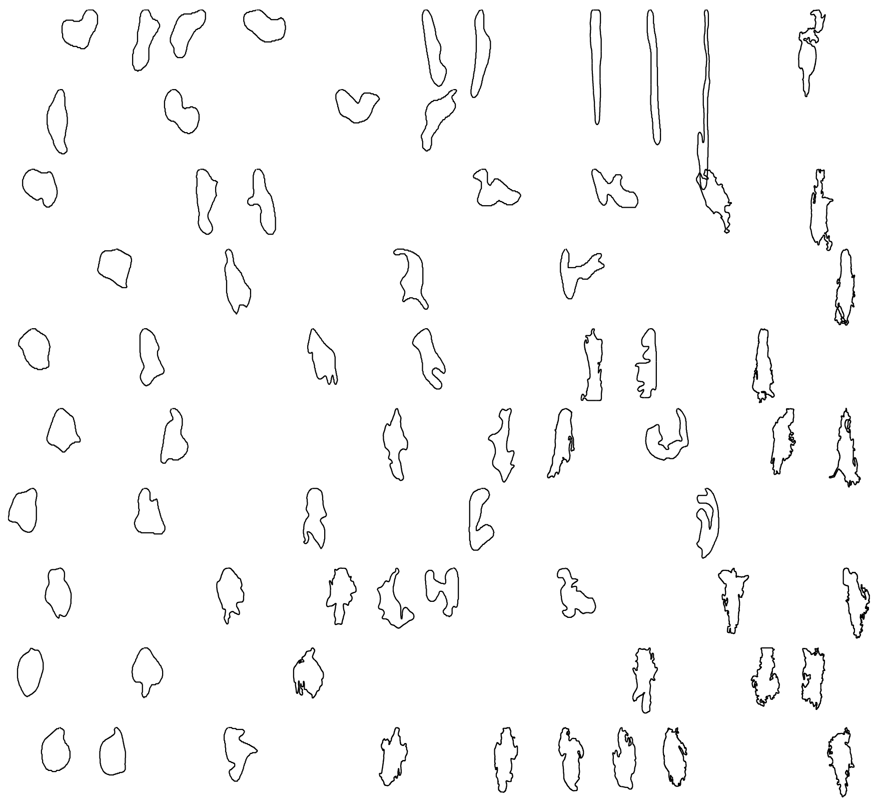

Figure 2: Sample contours on a Self-Organizing Map.

roughly three dimensions, elongation, smoothness and bending. This property is reflected also in the two remarkable covariances (*conv*,*comp*) and (*prax*,*cvar*) in Table 2.

There are many practical shortcuts that can be used to speed up the computation. For example, both $\mathcal{A}$ and $\mathcal{P}$ might have been calculated in a former stage of a process, and the computational effort for *comp* becomes negligible. The covariance matrix $\boldsymbol{C}$, which is needed in *prax* and in *evar*, has to be calculated only once. Thus, as far as calculational effort is considered, it is nearly equivalent to calculate just two or all the five descriptors. The authors recommend using all the five descriptors when little prior knowledge of applied shapes is available.

One can expect that the five descriptors apply also to more regular objects such as contours of tools or fauna. However, they might be of no use in the context of objects with loose or elastic parts such as with chromosomes, or in the context of objects with contours containing little information for recognition, as in the case of human faces.

5 Conclusions

The recognition power of five classical shape descriptors, which use only the contour of an object, was illustrated and discussed. The usefulness of the descriptors was pointed out in terms of computational effort, understandability, and cooperative power. The samples of the experiment were highly irregular, thus indicating general applicability of the discussed descriptor set. The future work involves comparing other shape descriptors to ones described here and experimenting with regular objects.

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